

COURSE SYLLABUS: CALCULUS BC

Eric Purvis

Email: Eric.Purvis@Jefferson.kyschools.us

Website: <http://www.dupontmanual.com/manualteacher/purvis/>

COURSE DESCRIPTION

Our school follows the AB block schedule with each class meeting for approximately 85 minutes on alternate days. The students who take the BC Calculus course are among our best mathematics students who have chosen our Math/Science/Technology mathematics course of study. This sequence of courses is designed to provide students with a challenging mathematical experience. These students will complete precalculus and differential calculus in their junior course. The text used in this class is Calculus I with Precalculus by Larson, Hostetler, Edwards. This is followed with the rest of the BC curriculum in their senior year using Calculus of a Single Variable also by Larson, et al. I have included the calculus portion of both courses in order to give a comprehensive look at our curriculum.

EXPECTATIONS:

Exams: Each exam has a non-calculator portion where students will demonstrate their ability to use techniques covered in the section and a calculator portion where they are expected to set up a mathematical model using calculus techniques then complete using calculator skills. The emphasis is always on correct notation, clear and concise communication of logic using mathematics and written explanations, demonstration of covered techniques, using calculus to model situations (with appropriate function, derivative, integral) and solve for requested information, justifying conclusions using appropriate mathematics, and using calculus skills to analyze what is occurring in a given situation (rate of change, net accumulation of change, graph analysis). Multiple choice and free response format questions similar to the AP test are included.

Calculator skills: All students own a TI 83-TI 89 calculator and are expected to learn how to perform the following: to graph a function (includes parametric and polar), trace a graph and use calculator functions to locate requested points on a graph (max/min, intervals where increasing/decreasing, solutions to equations/zeros of function), graph a numerical derivative, produce the graphs of a function and its derivatives / integral, do graphical analysis using values found from graphs of derivatives, find the numerical value of a definite integral, identify requested area under the curve of a function, find a regression equation for a set of data, produce a table of values to provide analysis, use a table to find the limit of a given expression, use calculator functions to find terms of sequence and the sum of a finite series.

Communication Skills: Students are expected to be able to produce mathematical models with correct notation. Course requirements include oral presentation of problems (from homework, exams, in class explorations) where the student will demonstrate correct notation, vocabulary, justification of

process/logic. During these presentations students are expected to answer any relevant questions from the class. They may also be asked questions designed to emphasize important details that are commonly forgotten, to generalize a process, to apply the concept to a different situation, and to clarify their explanation. Course work includes labs which have students investigate a situation, such as limit, using numerical values. They are asked to provide generalizations in a written format (example: situations where limits do not exist), to identify patterns and observed connections (example: like the connections between the values of derivatives and what is occurring in the graph of a function), and compare different approaches to a problem.

Grading:

Grades can be accessed from home utilizing the Parent Portal. For more information visit <http://www.jefferson.kyschools.us/Parent/ParentPortal/faqs.html>

$$G(x) = \begin{cases} A, & \text{if } 90 \leq x \\ B, & \text{if } 80 \leq x < 90 \\ C, & \text{if } 70 \leq x < 80 \\ D, & \text{if } 60 \leq x < 70 \\ U, & \text{if } x < 60 \end{cases}$$

Grade Breakdown:

Tests: 40%

End of unit exams will be given to assess students' summative understanding. Tests will incorporate both multiple choice and free response questions, simulating the style of the AP test.

Quizzes: 30%

Micro-quizzes will be administered on a regular basis to gauge understanding.

Homework: 15%

There will be a great deal of homework assigned for students to hone their skills. For the most part, homework is a way to practice new skills, just as in any sport. Homework will be broken down into core problems and challenges to promote student choice and mathematical blessings.

Classwork 15%

In class labs and activities are an important component as they will be the building blocks for understanding new calculus concepts. Activities involving teams, thought-provoking questions, and presentations are to be expected

Technology:

All students are required to have a graphing calculator (a full list of appropriate calculators can be found at http://www.collegeboard.com/student/testing/ap/calculus_ab/calc.html?calcab#list). Students must be able to use the graphing calculator to find zeros of a function/intersection points of two functions, coordinates of maximum or minimum values of a function, find a numeric derivative, and to evaluate a

definite integral. Students will use calculators to solve problems, interpret results, experiment, and to support conclusions.

The Dragon vs. Mathematical Righteousness

Do not look at the dragon!

Think of the gnarliest math problem you have encountered to date. Now imagine the difficulty tenfold and you get a sense for what might be awaiting you in this course. Big tests, challenging problems, hideous algebraic manipulations, the AP test in May—these are the dragon. If you stare into the teeth of the dragon, all hope is lost and you will cower in fear.

All dragons of lore have a weakness, and this is what you, the brave knights of calculus must attack. I encourage students to take up their swords of mathematics and slay the beasts that lay in ambush in the coming year. Find the weak spots in the problem and attack it. Perhaps another chink in the armor may present itself. Continue attacking piece by piece and, before long, you will have slain the dragon in the name of mathematical righteousness!

Stay vigilant and don't look at the dragon!

TOPIC OUTLINE FOR JUNIOR YEAR

Textbook: Calculus I with Precalculus: Larson, Hostetler, Edwards

Resources: Calculus of a Single Variable, Seventh Edition: Larson, Hostetler, Edwards

Limits and their Properties (5 class periods, 1 test)

- Tangent Line Problem
- Area Problem
- Review of distance, area, volume from geometry
- Graphical approach to limits
- Finding a limit numerically by constructing a table
- Limits that fail to exist (rt-lft disagreement, unbounded, oscillating)
- Formal definition of a limit
- Epsilon-Delta proofs
- Properties of limits
- Limits of functions (polynomial, rational, radical, composite, trig)
- Techniques (substitution, simplify then substitution, rationalizing)
- The Squeeze Theorem
- Limit of $\sin x / x$ and $(1 - \cos x) / x$
- Continuity at a point, on open interval, on the real number line (definition)
- Types of discontinuities
- One-sided limits and continuity on a closed interval
- Properties of continuity (scalar multiple, sum, difference, product, quotient)
- Continuity of a composite function
- Testing for continuity
- The Intermediate Value Theorem
- Infinite limit (definition)
- Vertical asymptotes
- Properties of infinite limits

Differentiation (10 class periods, 2 tests)

- Derivative of a function (definition)
- Techniques for finding the derivative with the limit definition
- Relationship between differentiability and continuity
- Conditions where the derivative does not exist
- Differentiation rules (constant, power, constant multiple, sum, difference, product, quotient, chain, general power rule, higher order derivatives)
- Derivatives of trigonometric functions
- Average rate of change vs. instantaneous rate of change
- Simplifying derivatives
- Implicit differentiation
- Finding tangent line equation

- Velocity and acceleration problems
- Related rates

Applications of Derivatives (10 class periods, 2 tests)

- Finding extrema of a function
- Absolute extrema vs. relative extrema
- Finding critical numbers
- Finding extrema on a closed interval
- Rolle's Theorem
- The Mean Value Theorem
- First Derivative Test
- Finding increasing/decreasing intervals for a function
- Determining concavity using the second derivative
- Points of inflection
- Limits at infinity (definition)
- Horizontal asymptotes
- Analysis of graphs of functions
- Applied optimization problems
- Linear approximation (Tangent line approximation)
- Differentials (definition)
- Propagated error
- Differential formulas (constant multiple, sum, difference, product, quotient)
- Leibnitz notation
- Differential of composite function
- Approximating function values using differentials

Exponential/Logarithmic Functions (6 class periods, 1 test)

- Inverse functions, their reflective property, conditions for the existence of an inverse function, guide to finding the inverse of a function, one-to-one functions
- Exponential functions, graphs, properties
- Natural exponential function
- Derivative of the natural exponential function
- Logarithmic function and properties
- Logarithmic expansion and rewriting expression as a single logarithm
- Logarithmic functions with different bases and the Change of Base formula
- The natural logarithmic function (defined as an integral), properties of the graph, properties of the function, the number e
- Derivative of the natural logarithmic function—including \ln of absolute value
- Logarithmic expansion as an aid to taking the derivative
- Exponential / logarithmic function with base a —definitions, properties of graphs, derivatives
- Solving equations which include exponential and logarithmic expressions
- Applications: growth/decay, compound interest

Trigonometric Functions (5 class periods, 1 test)

- Basic trigonometric functions as circular functions and in right triangle
- Graphs of trig functions
- Inverse trig functions and their graphs
- Solving equations which include trig and inverse trig expressions
- Derivatives of trigonometric and inverse trig functions
- Applications in situations which can be modeled by trigonometric functions
- Trig Identities

TOPIC OUTLINE FOR SENIOR YEAR

Textbook: Calculus of a Single Variable, Seventh Edition: Larson, Hostetler, Edwards

References/Resources:

Calculus, Fourth Edition: Larson, Hostetler, Edwards

Calculus, Concepts and Application: Paul A. Foerster

Resources for Calculus: A. Wayne Roberts

How to Prepare for the AP Calculus Advanced Placement Examination: Barron's

Cracking the AP Calculus AB and BC: The Princeton Review

Calculus, Fourth Edition: Schaum's Outlines

Released multiple choice and free response questions from The College Board

Member of Calculus listserv through The College Board

Review/Revisit: (4 class periods)

- Review of limits—finding limits using tables, graphs, algebraic techniques, circumstances where a limit doesn't exist, one sided limits
- Introduce L'Hopital's Rule using indeterminate forms zero/zero and infinity/infinity to allow review of derivatives, infinite limits
- Formal limit definition of derivative, basic rules of differentiation
- Review continuity and relationship between differentiation and continuity
- Application of differentiation: tangent/normal lines, optimization, curve analysis, related rates

Test: Review topics/L'Hopital's Rule

Chapter 4: Integration (9 class periods)

- Antiderivative (definition and representation)
- Solving differential equations
- Indefinite integration
- Basic integration rules (constant, constant multiplier, sum, difference, power rule, trigonometric functions)
- Techniques for integrating (polynomials, rewrite rational and/or trig expressions)
- Finding a particular solution
- Slope fields (how to draw, how to sketch function)
- Sigma notation and summation formulas
- Area of a plane region using limit approach
- Upper and lower sums and their limits
- Area of a region in a plane (limit definition)
- Finding area of a region using Riemann sum (rectangle approximation with horizontal and vertical orientation and regular partition of interval)
- Area using Midpoint formula
- Riemann sum (irregular partition definition)
- Norm of partition and its use
- Definite integrals (definition)
- Relationship between continuity and integrability

- Evaluating definite integral as a limit
- Evaluating definite integral using geometric shapes and formulas
- Properties of definite integrals (identical and reversed limits of integration, additive interval, constant multiplier, sum, difference)
- Preservation of inequality when integrating on closed interval

Test: Chapter 4.1-4.3

- The Fundamental Theorem of Calculus and its use
- Integration with absolute values
- The Mean Value Theorem for Integrals
- Average value of a function on an interval
- Integrals as a functions
- The Second Fundamental Theorem of Calculus
- Integration techniques (pattern recognition, formal change of variables including change of limits in definite integrals, general power rule, even and odd functions)
- Numerical integration--Trapezoid Rule, Simpson's Rule, error for both

Test: Chapter 4.4-4.6

Chapter 5: Log., exp., and other transcendental functions (11 class periods)

- The natural logarithmic function (defined as an integral), properties of the graph, properties of the function, the number e
- Derivative of the natural logarithmic function—including \ln of absolute value
- Logarithmic expansion as an aid to taking the derivative
- Integration of the natural logarithmic function
- Long division, u -substitution as an aid to integrating
- Integrals of the six basic trigonometric functions
- Inverse functions, their reflective property, conditions for the existence of an inverse function, guide to finding the inverse of a function, one-to-one functions
- Derivative of an inverse function using the derivative of the original function
- Relationship between the slopes of inverse functions at their inverse points
- Natural exponential function—definition as inverse of $\ln x$, operations, properties of the graph
- Derivative of the natural exponential function
- Integrals of the natural exponential function
- Exponential / logarithmic function with base a —definitions, properties of graphs, derivatives, integrals

Test: Chapter 5.1-5.5

- Power rule with real exponents
- Applications—tangent lines, rate of change, area, compound interest, exponential growth / decay
- Differential equations—solving by separation of variables, general and particular solutions, growth / decay models, Newton's Law of Cooling
- Inverse trigonometric functions—domain, range, graphs, properties

- Derivatives of inverse trigonometric functions
- Review of basic differentiation rules
- Integration involving inverse trigonometric functions including: change of variables and completing the square
- Review of basic integration rules

Test: Chapter 5.6-5.9

Chapter 6: Applications of Integration (3 class periods)

- Area of a region between two curves--using horizontal and vertical orientation, curves that intersect at more than two locations
- Integration as an accumulation process
- Volume of solids of revolution—disk, washer, shell, using a combination of several integrals to calculate, where axis of revolution is and is not a coordinate axis
- Volume calculated using known cross-sections
- Arc length and area of surface of revolution

Test: Chapter 6.1-6.4

Chapter 7: Integration (4 class periods)

- Review of basic integration
- Rewriting using trigonometric identities
- Integration by parts—including repeated use, tabular method
- Trigonometric integrals using trig identities to rewrite and use power rule
- Method of partial fractions—using partial fraction decomposition (linear and quadratic factors) and long division to rewrite in appropriate format

Test: Chapter 7.1, 7.2, 7.3, 7.5

Semester Final

Chapter 8: Infinite Series (7 class periods)

- Sequences—definition, limit of a sequence, properties of limits of sequences
- Squeeze Theorem for Sequences
- Absolute Value Theorem for finding the limit of a sequence
- Pattern recognition and finding the nth term of a sequence
- Monotonic and bounded sequences—definition, using to determine convergence
- Series—definition
- Convergence of series—definition of partial sums, conditions for convergence and divergence
- Geometric series—definition, convergence, sum of convergent infinite series, applications
- Properties of convergent infinite series—constant multiplier, sum, difference
- Limit of nth term of convergent series
- Nth term test for divergence
- The Integral Test, includes discussion that the first few terms may be excluded from an investigation of convergence

- P-Series Test, includes exploration of the harmonic series

Test: Chapter 8.1-8.3

- Direct Comparison Test, emphasis on the importance of the inequality being appropriate for convergence/divergence and the possible exclusion of the first few terms to establish this inequality
- Limit Comparison Test, emphasis that limit must be finite and positive
- Alternating Series Test, emphasis that decreasing nature must be established for all values of n and discussion of the alternating harmonic series, the Alternating Series Remainder, finding the number of terms necessary to approximate the sum within a required error margin
- Absolute and Conditional convergence—definition, how to establish, and importance to convergence if terms of series are rearranged
- Ratio and Root Test—definition and what to do if the test is inconclusive
- Strategies for using the various tests for convergence/divergence—what test to use for various functions, order in which to attempt to use tests

Test: Chapter 8.4-8.6

- Taylor and Maclaurin polynomials and approximations—how to find a Taylor polynomial for a function, comparison of function and Taylor approximations in a graphical setting
- Taylor's Theorem (LaGrange Error) and the remainder for a Taylor/Maclaurin polynomial approximation—how to find the number of terms necessary for a given level of accuracy, approximation and range of actual value using error value
- Power series—definition, finding the radius and interval of convergence including determining the convergence at the endpoints of the interval, differentiation and integration of power series including possible changes in the interval of convergence
- Representing functions by Power Series--how to find a power series for functions which are in the form of a sum for a infinite geometric series, operations with power series, finding a power series by integration, use of series to approximate function values and the value of π
- Taylor and Maclaurin Series—definition, form of a convergent power series, how to find a Taylor/Maclaurin series for a function and how to determine convergence by using the remainder, binomial series format, using a list of power series for elementary functions to find the power series for functions which are developed by using function operations including function composition

Test: Chapter 8.7-8.10

Chapter 9: Conics, Parametric Equations, and Polar Coordinates (6 class periods)

- Review of conic sections (parabolas, ellipses, hyperbolas) including standard form, eccentricity, important features and using them to obtain graph
- Plane curves and parametric equations—definition, changing format between parametric equations and rectangular equations including domain, graph of curve using parametric equations
- Parametric equations and Calculus—parametric form of the derivative; finding slope, concavity and tangent lines including two tangent lines at a point for cycloid; finding area of region, arc length and area of surface of revolution using parametric form

Test: Chapter 9.1-9.3

- Polar coordinates and polar graphs—discussion of polar plane, directed distance, coordinate conversion, graphing polar equations, slope in polar form, tangent lines (horizontal, vertical, at the pole), special polar graphs
- Area and arc length in polar coordinates—finding area in a polar region, area bounded by a single curve, area between two curves, arc length and area of surface of revolution of a polar curve

Test: Chapter 9.4-9.5

Additional Topics (1 class period)

- Slope fields—how to draw, how to use diagram to obtain graph of function, comparison of graphs of function and its derivatives
- Euler's Method—use to find approximate solution of a differential equation
- First order linear differential equations and their solution

Vector Valued Functions (4 class periods)

- Definition
- Plane and space curve representations
- Limit
- Continuity
- Derivative and properties—includes finding intervals where the curve is smooth
- Integration—definite and indefinite
- Velocity, acceleration, and speed—definition, finding values along a plane/space curve, finding position function, motion and position of a projectile

Test: Additional Topics and Vector Valued Functions

Topics for extended study after AP test

Applications in work, centroids, center of mass, fluid pressure and fluid force

Semester Final

TEST PREPARATION

Students receive weekly assignments which provide students with the opportunity to numerically and graphically investigate relationships leading to various calculus theorems, to revisit previously covered material, to provide experience with more challenging functions, to experience the format of the AP exam with multiple choice and free response examples like those that will be found on the exam. These assignments are graded for accuracy, notation, quality of work, clarity of explanation, and justification using mathematical reasoning and well written sentences. In addition, approximately ten class periods

are used for test preparation. Students are given practice exams which have been released by the College Board. This allows them to practice time management, pacing, and test taking techniques. The practice exams are graded using scoring rubrics which have been provided by the College Board. This gives students the feel for what is an acceptable format for an answer, requirement for justifying work, and performance necessary for “passing” the exam. Students are encouraged to begin reviewing for the AP exam in November (AB material) and continue review through the spring semester. “Cramming” for the exam a week or two before it is given is discouraged.

This will be a very difficult class because it is college-level calculus. We are in this together. I encourage students to seek help (peers, NHS tutoring, myself) if they feel they are lagging behind. You will be challenged, but embrace it—your mind is changing. If this stuff was easy, everybody would be taking it. Do not hesitate to seek help as I am always willing to lend aid—I love math and I love teaching it!

Note: Mr. Purvis reserves the right to alter layout of course as needed throughout the year. Any changes will be disseminated in a timely fashion to students and parents.