

AP Calculus AB Differential Equations and Exponential Growth Worksheet #1

- If $g'(x) = 2g(x)$ and $g(-1) = 1$, find $g(x)$
- Sketch a slope field for: a) $\frac{dy}{dx} = 2x$ b) $\frac{dy}{dx} = \frac{-x}{2}$ c) $\frac{dy}{dx} = 2y + 6$ d) $\frac{dy}{dx} = 2$
- For each of the differential equations in problem #2 find the particular solution if each curve contains $(0,1)$.

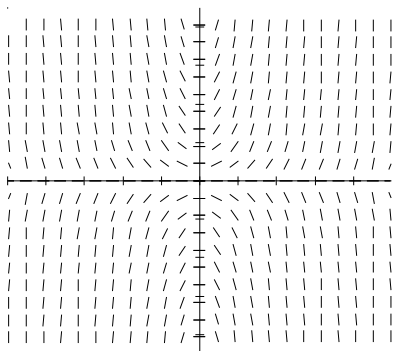
4-6. Match the differential equation with a slope field below.

a) $\frac{dy}{dx} = y^2$

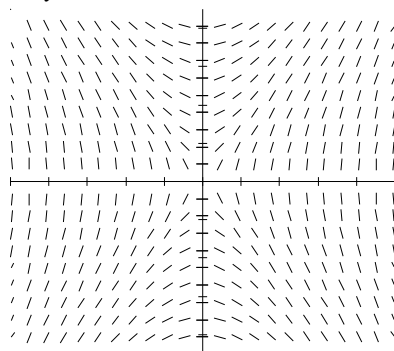
b) $\frac{dy}{dx} = \frac{2x}{y}$

c) $\frac{dy}{dx} = 2xy$

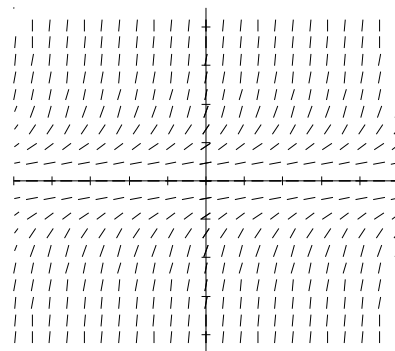
d) $\frac{dy}{dx} = x^2$



4.



5.



6.

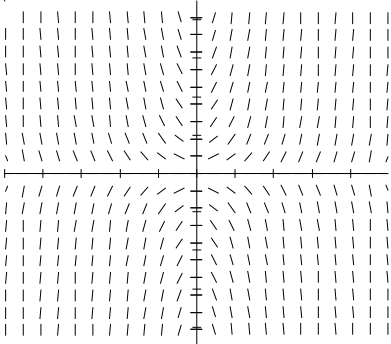
- At any time $t \geq 0$, in days, the rate of growth of a bacteria population is given by $y' = ky$, where k is a constant and y is the number of bacteria present. The initial population is 1,000 and the population triples during the first five days. a) Write an expression for y at any time $t \geq 0$. b) By what factor will the population have increased in the first ten days? c) At what time t , in days, will the population have increased by a factor of six?
- Find the particular solution that satisfies the initial condition.
 - $\frac{dy}{dx} = \frac{y}{2x}$ $(4,1)$
 - $\frac{dy}{dx} = \sqrt{xy}$ $(1,0)$
 - $y' = y$ $(2,3)$
- If $(g'(x))^2 = g(x)$ for all real x and $g(0)=0$, and $g(4)=4$, then $g(1)$?
- The concentration of a medicine injected into the bloodstream drops at a rate proportional to the existing concentration. If the factor of proportionality is 30 % per hour, in how many hours will the concentration be one-tenth of the initial concentration?
- A cup of coffee at temperature 180 degrees F is placed on a table in a room at 68 degrees F. The differential equation for its temperature at time t is $\frac{dy}{dx} = -0.11(y - 68)$. A) What is the temperature of the coffee in ten minutes? B) How long will it take the temperature of the coffee to reach 75 degrees F.?

ANSWERS

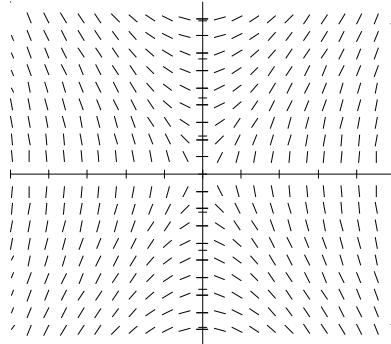
1. $g(x) = e^{2x+2}$

2.

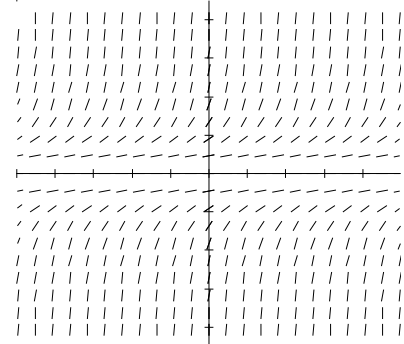
3. a) $y = x^2 + 1$ b) $y = -\frac{x^2}{4} + 1$ c) $y = 4e^{2x} - 3$ d) $y = 2x + 1$



4. c) $\frac{dy}{dx} = 2xy$



5. b) $\frac{dy}{dx} = \frac{2x}{y}$



6. a) $\frac{dy}{dx} = y^2$

7. a) $y = 1000(3^{\frac{t}{5}})$

b) 9

c) $t = 5 \log_3 6$

8. a) $y = \frac{\sqrt{x}}{2}$ b) $y = \frac{1}{9} \left(x^{\frac{3}{2}} - 1 \right)^2$ c) $y = 3e^{x-2}$

9. $g(1) = \frac{1}{4}$

10. $t = -\frac{10 \ln 10}{3} \text{ hrs}$

11. A) 105.282°
B) $t = 25.205 \text{ min}$