AP Calculus BC Differential Equations and Exponential Growth Worksheet

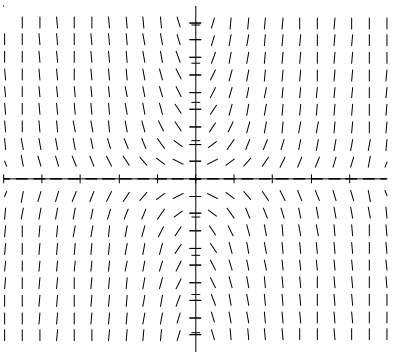
1. If $g'(x) = 2g(x)$ and $g(-1) = 1$, find $g(x)$

2. Sketch a slope field for: a) $\frac{dy}{dx} = 2x$ b) $\frac{dy}{dx} = \frac{-x}{2}$ c) $\frac{dy}{dx} = 2y + 6$ d) $\frac{dy}{dx} = 2$

3. For each of the differential equations in problem #2 find the particular solution if each curve contains (0,1).

4-6. Match the differential equation with a slope field below.

a) $\frac{dy}{dx} = y^2$ b) $\frac{dy}{dx} = \frac{2x}{y}$ c) $\frac{dy}{dx} = 2xy$ d) $\frac{dy}{dx} = x^2$



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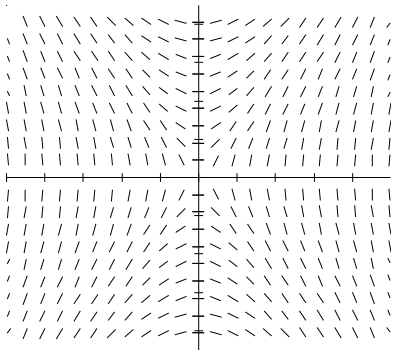

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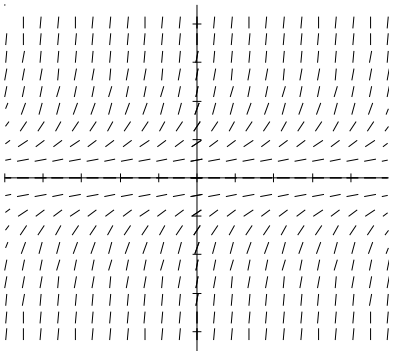


6.

7. At any time $t \geq 0$, in days, the rate of growth of a bacteria population is given by $y' = ky$, where k is a constant and y is the number of bacteria present. The initial population is 1,000 and the population triples during the first five days. a) Write an expression for y at any time $t \geq 0$. b) By what factor will the population have increased in the first ten days? c) At what time t, in days, will the population have increased by a factor of six?

8. Find the particular solution that satisfies the initial condition.
   a) $\frac{dy}{dx} = \frac{y}{2x}$ (4,1) b) $\frac{dy}{dx} = \sqrt{xy}$ (1,0) c) $y' = y$ (2,3)

9. If $(g'(x))^2 = g(x)$ for all real x and g(0)=0, and g(4)=4, then g(1)?

10. A conservation organization releases 25 Florida panthers into a game preserve. After two years, there are 39 panthers in the preserve. The Florida preserve has a carrying capacity of 200 panthers.

    A) Write a logistic equation that models the population of panthers in the preserve.

    B) Find the population after 5 years.

    C) At what time is the population of panthers growing most rapidly?

11. Let $y(t)$ be the temperature, in degrees Fahrenheit, of a cup of tea at time *t* minutes, $t \geq 0$. Room temperature is 70° and the initial temperature of tea is 180°. The tea's temperature at time *t* is described by the differential equation $\frac{dy}{dt} = -\frac{1}{10}(y - 70)$, with the initial condition $y(0) = 180$.

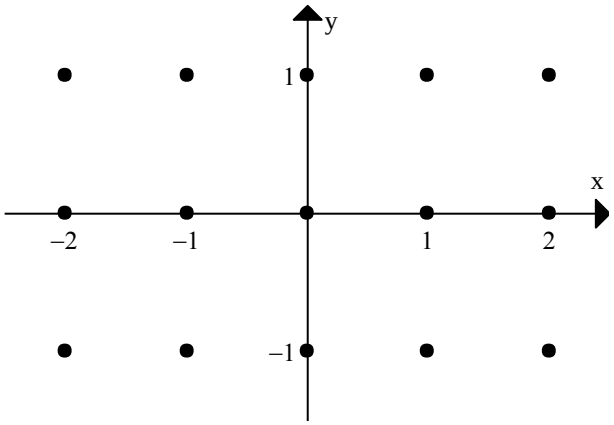
a.) Write an expression for $y(t)$, where *t* is measured in minutes.

b.) How hot is the tea after 10 minutes?

c.) If the tea is safe to drink when its temperature is less than 120°, at what time is the tea safe to drink?

12. Consider the differential equation $\frac{dy}{dx} = \frac{x + y}{2x}$.

a.) On the axes provided, sketch a slope field for the given differential equation at the twelve points indicated.



b.) Find the general solution of the given differential equation in terms of a constant *C*.

c.) Find the particular solution of the differential equation that satisfies the initial condition $y(1) = 3$.

d.) Use Euler's method to approximate y(1.75) starting at $y(1) = 3$ and using step size h= .25 .

13. Suppose that a student has three hours to cram for an AP Calculus AB exam and during this time wishes to memorize a set of 60 theorems and formulas. According to psychologists, the rate at which a person can memorize a set of facts is proportional to the number of facts remaining to be memorized. Thus if the student memorizes *y* facts in *t* minutes, $\frac{dy}{dt} = k(60 - y)$ where *k* is a positive constant and $y < 60$ for all $t \geq 0$. It is assumed that initially zero facts are memorized. The student is able to memorize 15 facts in the first 20 minutes.

a.) Write an expression for $y(t)$, where *t* is measured in minutes.

b.) How many formulas (not the nearest one)will the students memorize in one hour?

c.) How long will it take for the student to memorize half of the facts?

Answers

1. $y = e^{2x+2}$

2. look at winplot

3. a) $y = x^2 + 1$ b) $y = -\frac{1}{4}x^2 + 1$ c) $y = 4e^{2x} - 3$ d) $y = 2x + 1$

4. C 5. B 6. A

7. a) $y = 1000\left(3^{\frac{t}{5}}\right)$ b) 9 c) 8.155 days

8. a) $y = \frac{\sqrt{x}}{2}$) $y = \left(\frac{1}{3}x^{\frac{3}{2}} - \frac{1}{3}\right)^2$ c) $y = 3e^{x-2}$

9. $y = \frac{x^2}{4}$

10. a.) $p(t) = \frac{200}{1 + 7e^{-0.264t}}$ b.) 70 c.) @ 7.371 years

11. a.) $y(t) = 110e^{-\frac{t}{10}} + 70$ b.) 110.467 degrees F c.) 7.885 min

12. b.) $y = x + C\sqrt{x}$ c.) $y = x + 2\sqrt{x}$ d.) 4.431

13. a.) $y(t) = 60 - 60\left(\frac{3}{4}\right)^{\frac{t}{20}}$ b.) 35 c.) 48.188 minutes