

AP Calculus Chapter 7 Fun Worksheet ☺

1. Let the first quadrant region enclosed by the graph of  $y = \frac{1}{x}$  and the lines  $x = 1$  and  $x = 4$  be the base of a solid. If cross sections perpendicular to the  $x$ -axis are semicircles (inscribed by its diameter), the volume of the solid is

- (A)  $\frac{3\pi}{64}$  units<sup>3</sup>      (B)  $\frac{3\pi}{32}$  units<sup>3</sup>      (C)  $\frac{3\pi}{16}$  units<sup>3</sup>      (D)  $\frac{3\pi}{8}$  units<sup>3</sup>      (E)  $\frac{3\pi}{4}$  units<sup>3</sup>

2. Let  $R$  be the region in the first quadrant bounded by the  $x$ -axis and the curve  $y = 2x - x^2$ . The volume produced when  $R$  is revolved about the  $x$ -axis is

- (A)  $\frac{16\pi}{15}$  units<sup>3</sup>      (B)  $\frac{8\pi}{3}$  units<sup>3</sup>      (C)  $\frac{4\pi}{3}$  units<sup>3</sup>      (D)  $16\pi$  units<sup>3</sup>      (E)  $8\pi$  units<sup>3</sup>

3. The region in the first quadrant bounded above by the graph of  $y = \sqrt{x}$  and below by the interval  $[0, 4]$  is revolved about the  $x$ -axis. If a plane perpendicular to the  $x$ -axis at the point where  $x = k$  divides the solid into parts of equal volume, then  $k =$

- (A) 2.77      (B) 2.80      (C) 2.83      (D) 2.86      (E) 2.89

4. The region in the first quadrant bounded by the graph of  $y = \text{Arcsin } x$ ,  $y = \frac{\pi}{2}$ , and the  $y$ -axis is rotated about the  $y$ -axis. The volume of the solid generated is given by

- (A)  $\pi \int_0^{\pi/2} y^2 dy$       (B)  $\pi \int_0^1 (\text{Arcsin } x)^2 dx$       (C)  $\pi \int_0^{\pi/2} (\text{Arcsin } x)^2 dx$       (D)  $\pi \int_0^{\pi/2} (\sin y)^2 dy$       (E)  $\pi \int_0^1 (\sin y)^2 dy$

5. Let  $R$  be the region in the first quadrant enclosed by the lines  $x = 0$  and  $y = 2$  and the graph of  $y = e^x$ . The volume of the solid generated when  $R$  is revolved about the  $x$ -axis is given by

- (A)  $\pi \int_0^2 (4 - e^{2x}) dx$       (B)  $\pi \int_0^{\ln 2} (2 - e^x)^2 dx$       (C)  $2\pi \int_0^{\ln 2} x(2 - e^x) dx$       (D)  $\pi \int_0^{\ln 2} (4 - e^{2x}) dx$       (E)  $2\pi \int_0^2 x(2 - e^x) dx$

6. Let  $R$  be the region bounded by the graph of  $f(x) = e^{\frac{x}{2}}$ , the  $x$ -axis,  $y$ -axis, and the line  $x = 4$ . Set up an integral or sum of integrals (but do NOT evaluate) that will find the volume of the solid if  $R$  is revolved about:

- the  $x$ -axis
- the line  $y = -2$
- the line  $y = 12$
- the  $y$ -axis
- the line  $x = -1$
- the line  $x = 4$
- the line  $x = 5$

7. Set up an integral (but do NOT evaluate) that will find the volume of the solid whose base is the region  $R$ . Let  $R$  be the region bounded by the graph  $f(x) = e^{\frac{x}{2}}$ , the  $x$ -axis,  $y$ -axis, and the line  $x = 4$  and whose cross sections perpendicular to the  $x$ -axis are:
- squares whose side is on  $R$
  - circles whose diameter is on  $R$
  - semicircles whose diameter is on  $R$
  - rectangles where the base (on  $R$ ) is twice the height.
  - equilateral triangles whose side is on  $R$

**#8-11: Find the volume of each region when it is revolved around the indicated axis.**

8. Region bounded  $y = 2x$ ,  $x = 0$ ,  $y = 4$ , revolved about:
- $x$ -axis
  - $y = 4$
  - $y = 6$
  - $y = -2$
9. Region bounded by  $y = x^2 - 4x + 5$  and  $y = 5 - x$ , revolved about
- $x$ -axis
  - $y = 8$
  - $y = -1$
10. Region bounded by  $y = x^2 + 1$  and  $y = x + 3$ , revolved about
- Find the area of the region
  - Find the volume of the solid formed by revolving the region  $R$  about the  $x$ -axis.
  - Find the volume of the solid formed by revolving the region  $R$  about the line  $y = 6$
  - Find the volume of the solid formed by revolving the region  $R$  about the line  $y = -2$
11. Let  $R$  be the region bounded by  $y = 4 - x^2$ ,  $y = 3x$  and the  $y$ -axis.
- Find the area of the region.
  - Find the volume of the solid formed by revolving the region  $R$  about the  $x$ -axis.

## Answer Sheet

1. B      2. A      3. C      4. D      5. D

6. a)  $\pi \int_0^4 (e^x) dx$       b)  $\pi \int_0^4 \left( (e^{\frac{x}{2}} + 2)^2 - 4 \right) dx$       c)  $\pi \int_0^4 \left( 144 - (12 - e^{\frac{x}{2}})^2 \right) dx$       d)  $2\pi \int_0^4 x e^{\frac{x}{2}} dx$

e)  $2\pi \int_0^4 (x+1)e^{\frac{x}{2}} dx$       f)  $2\pi \int_0^4 (4-x)e^{\frac{x}{2}} dx$       g)  $2\pi \int_0^4 (5-x)e^{\frac{x}{2}} dx$

7. a)  $\int_0^4 (e^x) dx$       b)  $\frac{\pi}{4} \int_0^4 (e^x) dx$       c)  $\frac{\pi}{8} \int_0^4 (e^x) dx$       d)  $\frac{1}{2} \int_0^4 (e^x) dx$       e)  $\frac{\sqrt{3}}{4} \int_0^4 (e^x) dx$

8. a)  $\frac{64\pi}{3}$       b)  $\frac{32\pi}{3}$       c)  $\frac{80\pi}{3}$       d.)  $\frac{112\pi}{3}$

9. a)  $\frac{117\pi}{5}$       b)  $\frac{243\pi}{5}$       c)  $\frac{162\pi}{5}$

10. a)  $\frac{9}{2}$       b)  $\frac{117\pi}{5}$       c)  $\frac{153\pi}{5}$       d.)  $\frac{207\pi}{5}$

11. a)  $\frac{13}{6}$       b)  $\frac{158\pi}{15}$