Finding "Impossible" Integrals - Classwork

Example: Let $F(x)$ be an antiderivative of $\dfrac{x}{x^3+1}$. If $F(0)=2$, find $F(3)$.

Solution: We need to take the integral of $\dfrac{x}{x^3+1}$. But can we? *u*-substitution does not work. We cannot split the function. There seems to be no possibility for inverse-trig. So how can we do the problem?

We are not being asked to find the indefinite integral of the expression. We just need to find the value of the integral function of this expression at 3.

So, if $F(x)=\int \dfrac{x}{x^3+1}\,dx$, then $F(3)-F(0)=\int_0^3 \dfrac{x}{x^3+1}\,dx$ and thus $F(3)=2+\int_0^3 \dfrac{x}{x^3+1}\,dx$.

Using the calculator: $F(3)=2+.879=2.879$.

2) Let $F(x)$ be an antiderivative of $\sqrt{4x^2+5}$. If $F(1)=3$, find $F(5)$.

$$F(5)-F(1)=\int_1^5 \sqrt{4x^2+5}\,dx$$

$$F(5)=F(1)+\int_1^5 \sqrt{4x^2+5}\,dx=3+25.866=28.866$$

3. Let $F(x)$ be an antiderivative of e^{x^2}. If $F(-1)=2$, find $F(-2)$.

$$F(-1)-F(-2)=\int_{-2}^{-1} e^{x^2}\,dx$$

$$F(-2)=F(-1)-\int_{-2}^{-1} e^{x^2}\,dx=2-14.990=-12.990$$

Finding "Impossible" Integrals - Homework

1. Let $F(x)$ be an antiderivative of $\dfrac{5x-2}{8x^4+2}$. If $F(1)=3$, find $F(6)$.

$$F(6)-F(1)=\int_1^6 \frac{5x-2}{8x^4+2}\,dx \Rightarrow F(6)=F(1)+\int_1^6 \frac{5x-2}{8x^4+2}\,dx=3+.206=3.206$$

2. Let $F(x)$ be an antiderivative of $\sqrt[3]{5x^2-4}$. If $F(2)=7$, find $F(5)$.

$$F(5)-F(2)=\int_2^5 \sqrt[3]{5x^2-4}\,dx \Rightarrow F(5)=F(2)+\int_2^5 \sqrt[3]{5x^2-4}\,dx=7+11.447=18.447$$

3. Let $F(x)$ be an antiderivative of $\sin^3 x$. If $F(-1)=4$, find $F(4)$

$$F(4)-F(-1)=\int_{-1}^4 \sin^3 x\,dx \Rightarrow F(4)=F(-1)+\int_{-1}^4 \sin^3 x\,dx=4+1.048=5.048$$

4. Let $F(x)$ be an antiderivative of $\ln\left(x^2+4x+12\right)$. If $F(10)=-2$, find $F(-1)$

$$F(10)-F(-1)=\int_{-1}^{10} \ln\left(x^2+4x+12\right)dx \Rightarrow F(-1)=F(10)-\int_{-1}^{10} \ln\left(x^2+4x+12\right)dx=-2-41.743=-43.743$$