

CALCULUS BC WORKSHEET ON LOGISTIC GROWTH

Work the following on **notebook paper**. Do not use your calculator.

1. Suppose the population of bears in a national park grows according to the logistic differential

equation $\frac{dP}{dt} = 5P - 0.002P^2$, where P is the number of bears at time t in years.

(a) Given $P(0) = 100$.

(i) Find $\lim_{t \rightarrow \infty} P(t)$.

(ii) What is the range of the solution curve?

(iii) For what values of P is the solution curve increasing? Decreasing? Justify your answer.

(iv) Find $\frac{d^2P}{dt^2}$ and use it to find the values of P for which the solution curve is concave up and concave down. Justify your answer.

(v) Does the solution curve have an inflection point? Justify your answer.

(vi) Use the information you found to sketch the graph of $P(t)$.

(b) Given $P(0) = 1500$.

(i) Find $\lim_{t \rightarrow \infty} P(t)$.

(ii) What is the range of the solution curve?

(iii) For what values of P is the solution curve increasing? Decreasing? Justify your answer.

(iv) For what values of P is the solution curve concave up? Concave down? Justify your answer.

(v) Does the solution curve have an inflection point? Justify your answer.

(vi) Use the information you found to sketch the graph of $P(t)$.

(c) Given $P(0) = 3000$.

(i) Find $\lim_{t \rightarrow \infty} P(t)$.

(ii) What is the range of the solution curve?

(iii) For what values of P is the solution curve increasing? Decreasing? Justify your answer.

(iv) For what values of P is the solution curve concave up? Concave down? Justify your answer.

(v) Does the solution curve have an inflection point? Justify your answer.

(vi) Use the information you found to sketch the graph of $P(t)$.

(d) How many bears are in the park when the population of bears is growing the fastest?

2. Suppose a rumor is spreading through a dance at a rate modeled by the logistic differential

equation $\frac{dP}{dt} = P \left(3 - \frac{P}{2000} \right)$. What is $\lim_{t \rightarrow \infty} P(t)$? What does this number represent in

the context of this problem?

TURN->>>

3. (From the 1998 BC Multiple Choice)

The population $P(t)$ of a species satisfies the logistic differential equation

$$\frac{dP}{dt} = P \left(2 - \frac{P}{5000} \right),$$

where the initial population is $P(0) = 3000$ and t is the time in years. What

is $\lim_{t \rightarrow \infty} P(t)$?

- (A) 2500 (B) 3000 (C) 4200 (D) 5000 (E) 10,000

4. Suppose a population of wolves grows according to the logistic differential

equation $\frac{dP}{dt} = 3P - 0.01P^2$, where P is the number of wolves at time t in

years. Which of the following statements are true?

I. $\lim_{t \rightarrow \infty} P(t) = 300$

II. The growth rate of the wolf population is greatest at $P = 150$.

III. If $P > 300$, the population of wolves is increasing.

- (A) I only (B) II only (C) I and II only (D) II and III only (E) I, II, and III

5. Suppose that a population develops according to the logistic equation

$$\frac{dP}{dt} = 0.05P - 0.0005P^2$$

where t is measured in weeks.

- (a) What is the carrying capacity?
 (b) A slope field for this equation is shown at the right.

Where are the slopes close to 0?

Where are they largest?

Which solutions are increasing?

Which solutions are decreasing?

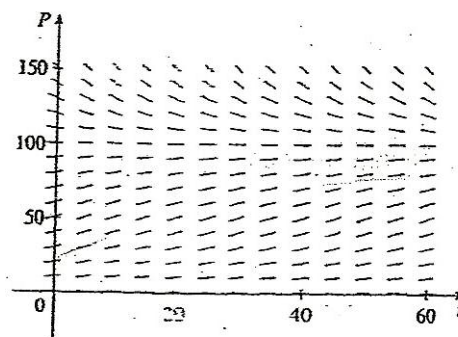
- (c) Use the slope field to sketch solutions for initial populations of 20, 60, and 120.

What do these solutions have in common?

How do they differ?

Which solutions have inflection points?

At what population level do they occur?



6. (a) On the slope field shown on the right

for $\frac{dP}{dt} = 3P - 3P^2$, sketch three

solution curves showing different types of behavior for the population P .

- (b) Describe the meaning of the shape of the solution curves for the population.

Where is P increasing?

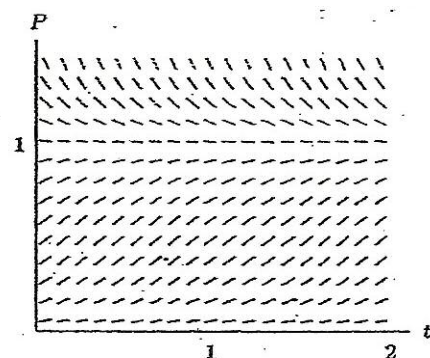
Decreasing?

What happens in the long run?

Are there any inflection points?

Where?

What do they mean for the population?



CALCULUS BC WORKSHEET 2 ON LOGISTIC GROWTH

Work the following on **notebook paper**. Use your calculator on 2(b) and (c), 3(c) and (d), 4(b) and (c), and 5(c) and (d) only.

1. Suppose you are in charge of stocking a fish pond with fish for which the rate of population

growth is modeled by the differential equation $\frac{dP}{dt} = 8P - 0.02P^2$.

(a) Given $P(0) = 50$.

(i) Find $\lim_{t \rightarrow \infty} P(t)$.

(ii) What is the range of the solution curve?

(iii) For what values of P is the solution curve increasing? Decreasing? Justify your answer.

(iv) Find $\frac{d^2P}{dt^2}$ and use it to find the values of P for which the solution curve is concave up and concave down. Justify your answer.

(v) Does the solution curve have an inflection point? Justify your answer.

(vi) Use the information you found to sketch the graph of $P(t)$.

(b) Given $P(0) = 300$.

(i) Find $\lim_{t \rightarrow \infty} P(t)$.

(ii) What is the range of the solution curve?

(iii) For what values of P is the solution curve increasing? Decreasing? Justify your answer.

(iv) For what values of P is the solution curve concave up? Concave down? Justify your answer.

(v) Does the solution curve have an inflection point? Justify your answer.

(vi) Use the information you found to sketch the graph of $P(t)$.

(c) Given $P(0) = 500$.

(i) Find $\lim_{t \rightarrow \infty} P(t)$.

(ii) What is the range of the solution curve?

(iii) For what values of P is the solution curve increasing? Decreasing? Justify your answer.

(iv) For what values of P is the solution curve concave up? Concave down? Justify your answer.

(v) Does the solution curve have an inflection point? Justify your answer.

(vi) Use the information you found to sketch the graph of $P(t)$.

2. A population of animals is modeled by a function P that satisfies the logistic differential

equation $\frac{dP}{dt} = 0.01P(100 - P)$, where t is measured in years.

(a) If $P(0) = 20$, solve for P as a function of t .

(b) Use your answer to (a) and your graphing calculator to find P when $t = 3$ years.

(c) Use your answer to (a) and your graphing calculator to find t when $P = 80$ animals.

TURN->>>

3. The rate at which a rumor spreads through a high school of 2000 students can be modeled by the differential equation $\frac{dP}{dt} = 0.003P(2000 - P)$, where P is the number of students who have heard the rumor t hours after 9AM.

- How many students have heard the rumor when it is spreading the fastest? Justify your answer.
- If $P(0) = 5$, solve for P as a function of t .
- Use your answer to (b) and your graphing calculator to determine how many hours have passed when half the student body has heard the rumor.
- Use your answer to (b) and your graphing calculator to determine how many students have heard the rumor after 2 hours.

4. A certain national park is known to be capable of supporting no more than 100 grizzly bears. Ten bears are in the park at present. The population growth of bears can be modeled by the logistic differential equation $\frac{dP}{dt} = 0.1P - 0.001P^2$, where t is measured in years.

- Solve for P as a function of t .
- Use your solution to (a) and your graphing calculator to find the number of bears in the park when $t = 3$ years.
- Use your solution to (a) and your graphing calculator to find how many years it will take for the bear population to reach 50 bears.

5. Suppose a rumor is spreading at a dance attended by 200 students. The rumor is spreading at a rate that is directly proportional to both the number of students who have heard the rumor and the number of students who have not heard the rumor. Let P be the number of students who have heard the rumor, and let t be the time in minutes since the rumor began to spread.

- Write a differential equation to model this rate of change.
- If $P(0) = 10$ and $P(15) = 50$, solve for P as a function of t .
- Use your solution to (b) to find the number of students who have heard the rumor after 1 hour.
- Use your solution to (b) to find the time it takes for 175 students to hear the rumor.

Answers to Worksheet 1 on Logistic Growth

1. (a) (i) 2500
(ii) [100, 2500)
(iii) increasing for [100, 2500)
(iv) concave up for (100, 1250) and concave down for (1250, 2500)
(v) yes, IP when $P = 1250$
(vi) sketch
 - (b) (i) 2500
(ii) [1500, 2500)
(iii) increasing for [1500, 2500)
(iv) concave down for (1500, 2500)
(v) no
(vi) sketch
 - (c) (i) 2500
(ii) (2500, 3000]
(iii) decreasing for (2500, 3000]
(iv) concave up for (2500, 3000)
(v) no
(vi) sketch
2. 6000; there are 6000 people at the dance.
3. E
4. C
5. (a) 100
(b) Close to 0? $P = 0$ and $P = 100$
Largest? $P = 50$
Increasing? $P(0) < 100$
Decreasing? $P(0) > 100$
- (c) In common? All have a limit of 100.
Differ? Two are increasing; one is decreasing.
Inflection points? The one with initial condition of 20.
At what pop. level does the inflection point occur? When $P = 50$.
6. (a) sketch
(b) Increasing? $P(0) < 1$
Decreasing? $P(0) > 1$
In the long run? $\lim_{t \rightarrow \infty} P(t) = 1$
Any inflection points? Yes
Where? When $P(0) = 0.5$
What do they mean for the population? The population is growing the fastest when $P(0) = 0.5$.

Answers to Worksheet 2 on Logistic Growth

1. (a) (i) 400
(ii) [50, 6000)
(iii) increasing for [50, 400)
(iv) concave up for (50, 200) and concave down for (200, 400)
(v) yes, IP when $P = 200$
(vi) sketch
- (b) (i) 400
(ii) [300, 6000)
(iii) increasing for [300, 400)
(iv) concave down for (300, 400)
(v) no
(vi) sketch
- (c) (i) 400
(ii) (400, 500]
(iii) decreasing for (400, 500]
(iv) concave up for (400, 500)
(v) no
(vi) sketch

2. (a) $P = \frac{100e^t}{e^t + 4}$ or $P = \frac{100}{1 + 4e^{-t}}$
(b) 83.393 animals
(c) 2.773 years

3. (a) 1000 students
(b) $P = \frac{2000e^{6t}}{e^{6t} + 399}$ or $P = \frac{2000}{1 + 399e^{-6t}}$
(c) 0.998 hours
(d) 1995.1089... so 1995 people

4. (a) $P = \frac{100e^{0.1t}}{e^{0.1t} + 9}$ or $P = \frac{100}{1 + 9e^{-0.1t}}$
(b) 13.042... or 13 bears
(c) 21.972 years

5. (a) $\frac{dP}{dt} = kP(200 - P)$

(b) $P = \frac{200e^{200kt}}{19 + e^{200kt}}$ or $P = \frac{200}{19e^{-200kt} + 1}$ where $k = \frac{\ln\left(\frac{19}{3}\right)}{3000}$