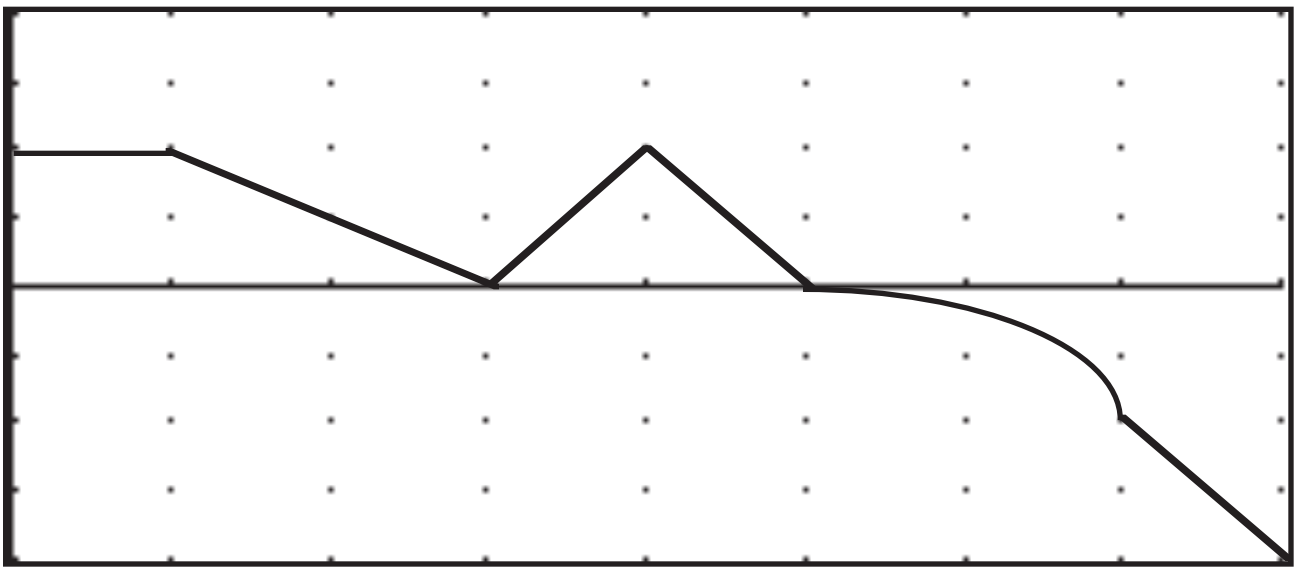
The Accumulation Function - Classwork

Let $F(x) = \int_0^x f(t)\,dt$ where the graph of $f(x)$ is below. Remember $f(x)$ is the same thing as $f(t)$.

Think of $f(x)$ as the rate of snowfall over a period of time. For instance, at $x = 1$, snow is falling at 2 inches per hour, at $x = 3$, it is not snowing, and at $x = 8$, snow is melting at 4 inches per hour.



1. Let $F(x) = \int_0^x f(t)\,dt$ where the graph of $f(x)$ is above (the graph consists of lines and a quarter circle)

a. Complete the chart

x	0	1	2	3	4	5	7	8
$F(x)$	0	2	3.5	4	5	6	$2+\pi$	$\pi-1$
$F'(x)$	2	2	1	0	2	0	-2	-4

b. On what subintervals of [0, 8] is F increasing? Decreasing? Justify your answer.

Increasing where $\boxed{F' > 0 : (0,3) \cup (3,5)}$ Decreasing where $\boxed{F' < 0 : (5,8)}$

c. Where in the interval [0, 8] does F achieve its minimum value? What is the minimum value? Justify answer.

$\boxed{0 \text{ at } x = 0}$

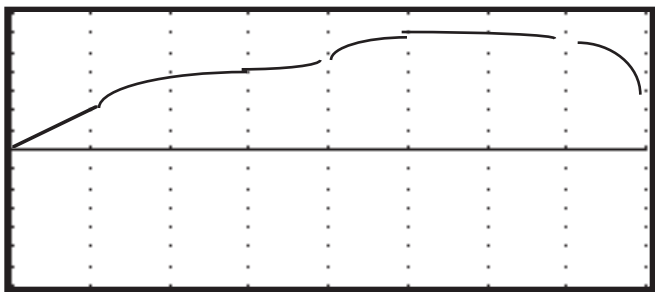
d. Where in the interval [0, 8] does F achieve its maximum value? What is the maximum value? Justify answer.

$\boxed{6 \text{ at } x = 5}$

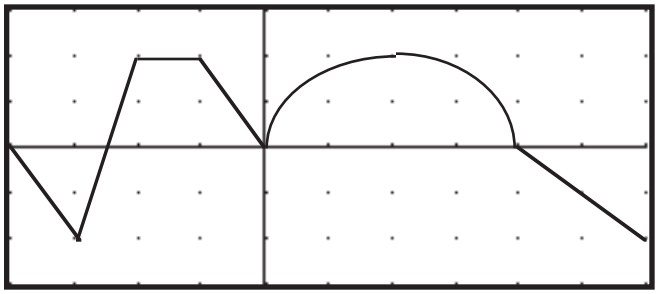
e. Find the concavity of F and any inflection points. Justify answers.

Up where $\boxed{F' \text{ increasing} : (3,4)}$ Down where $\boxed{F' \text{ decreasing} : (1,3) \cup (4,8)}$

f. Sketch a rough graph of $F(x)$



Example 2) Let $F(x) = \int_0^x f(t)\,dt$ where f is the function graphed below (consisting of lines and a semi-circle)



Find the following:

a) $\boxed{F(0) = 0}$ b) $\boxed{F(2) = \pi}$ c) $\boxed{F(4) = 2\pi}$ d) $\boxed{F(6) = 2\pi - 2}$

e) $\boxed{F(-1) = -1}$ f) $\boxed{F(-2) = -3}$ g) $\boxed{F(-3) = -3}$ h) $\boxed{F(-4) = -2}$

i) $\boxed{F'(4) = 0}$ j) $\boxed{F'(2) = 2}$ k) $\boxed{F'(6) = -2}$ l) $\boxed{F'(-3) = -2}$

m) On what subintervals of $[-4,6]$ is F increasing and decreasing. Justify your answer.
increasing where $\boxed{F' > 0 : (-2.5,0) \cup (0,4)}$ decreasing where $\boxed{F' < 0 : (-4,-2,5) \cup (4,6)}$

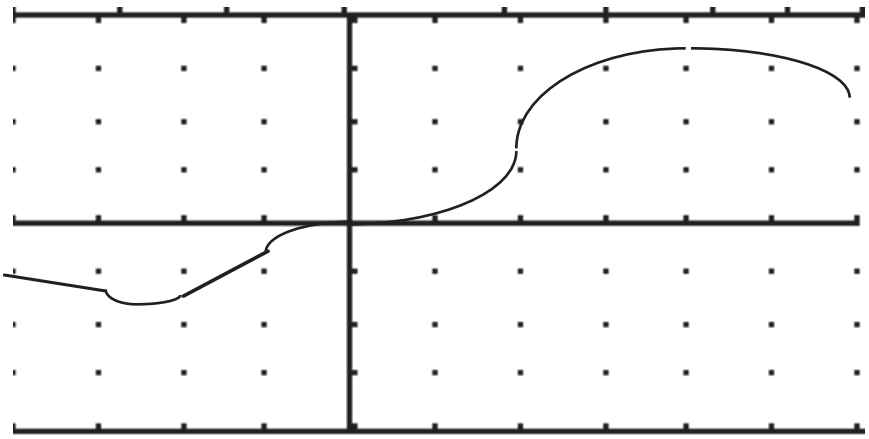
n) Where in the interval $[-4,6]$ is F achieve its minimum value? What is the minimum value?
$\boxed{-3.5 \text{ at } x = -2.5}$

o) Where in the interval $[-4,6]$ does F achieve its maximum value? What is the minimum value?
$\boxed{2\pi \text{ at } x = 4}$

p) Where on the interval $[-4,6]$ is F concave up? Concave down? Justify your answer.
Concave up where F' is increasing: $(-3,-2) \cup (0,2)$, Down where F' is decreasing $\boxed{(-4,-3) \cup (-1,0) \cup (2,6)}$

q) Where does F have points of inflection? $\boxed{x = -3, x = 0, x = 2}$

r) Sketch the function F



Assume each tic mark on the y-axis is 2 units