$\boxed{BC}$

$\boxed{A1}$ $\lim_{x\to 0} \dfrac{e^x + \cos x - x - 2}{x^4 - x^3} = \frac{0}{0} \overset{L'H}{=} \lim_{x\to 0} \dfrac{e^x - \sin x - 1}{4x^3 - 3x^2} = \frac{1-0-1}{0-0} = \frac{0}{0} \overset{L'H}{=} \lim_{x\to 0} \dfrac{e^x - \cos x}{12x^2 - 6x} = \frac{1-1}{0}$

$= \lim_{x\to 0} \dfrac{e^x + \sin x}{24x - 6} = \frac{1}{-6} = \boxed{-\frac{1}{6}}$ $\boxed{C}$

$\boxed{A2}$ $\lim_{x\to 1^+} (x^2-1)\ln(x-1)$ $\quad 0 \cdot -\infty$?

$= \lim_{x\to 1^+} \dfrac{\ln(x-1)}{\frac{1}{x^2-1}} = \frac{-\infty}{\infty}$ $(x^2-1)^{-1}$ $\overset{L'H}{=} \lim_{x\to 1^+} \dfrac{\frac{1}{x-1}}{\frac{-2x}{(x^2-1)^2}} = \lim_{x\to 1^+} \dfrac{(x^2-1)^2}{(x-1)(-2x)} = \lim_{x\to 1} \dfrac{(x+1)^2(x-1)^{\cancel{2}}}{\cancel{(x-1)}(-2x)} = \frac{0}{-2} = \boxed{0}$

$\boxed{A}$

$\boxed{A3}$ $\lim_{x\to 2} \dfrac{\int_2^x e^{t^2-1}\,dt}{x^3 - 4x} = \frac{0}{0} \overset{L'H}{=} \lim_{x\to 2} \dfrac{e^{x^2-1}}{3x^2 - 4} = \boxed{\dfrac{e^3}{8}}$ $\boxed{C}$

$\boxed{A4}$ $x(t) = \cos t$
$y(t) = \sin^2(4t)$

$\lim_{t\to 0} \dfrac{dy}{dx}$

well $\dfrac{dy}{dx} = \dfrac{dy}{dt} \cdot \dfrac{dt}{dx}$ ← reciprocal of $\frac{dx}{dt}$

$= 2\sin(4t)\cdot\cos(4t)\cdot 4 \cdot \left(-\frac{1}{\sin t}\right)$

$= -\dfrac{8\sin(4t)\cos(4t)}{\sin t}$ double angle

$\dfrac{dy}{dx} = \dfrac{-4\sin(8t)}{\sin t}$

$\lim_{t\to 0} \dfrac{-4\sin(8t)}{\sin t} \overset{L'H}{=} \dfrac{-4\cos(8t)\cdot 8}{\cos t} = \dfrac{-4(1)(8)}{1} = \boxed{-32}$

$\boxed{A}$

$\boxed{A5}$ $P(t) = 500\left(1+\frac{2}{t}\right)^t$

$\lim_{t\to\infty} 500\left(1+\frac{2}{t}\right)^t = 500 \lim_{t\to\infty}\left(1+\frac{2}{t}\right)^t = L$

$= \ln \lim_{t\to\infty}\left(1+\frac{2}{t}\right)^t = \ln\left(\frac{L}{500}\right)$

$\lim_{t\to\infty} \ln\left(1+\frac{2}{t}\right)^t = \ln\left(\frac{L}{500}\right)$

$\lim_{t\to\infty} t\ln\left(1+\frac{2}{t}\right) = \ln\frac{L}{500}$

$\lim_{t\to\infty} \dfrac{\ln\left(1+\frac{2}{t}\right)}{\frac{1}{t}} = \ln\frac{L}{500}$

$L'H$ $\lim_{t\to\infty} \dfrac{\left(\frac{1}{1+\frac{2}{t}}\right)\frac{-2}{t^2}}{\frac{-1}{t^2}} = \ln\frac{L}{500}$

$\lim_{t\to\infty} \left(\frac{1}{1+\frac{2}{t}}\right)\frac{+2}{\cancel{t^2}}\cdot\frac{\cancel{t^2}}{+1} = \ln\frac{L}{500}$

$e^2 = e^{\ln\frac{L}{500}}$

$e^2 = \frac{L}{500}$

$\boxed{500e^2 = L}$

$\boxed{E}$

[B1]

(6) $\int 2x \cos 4x \, dx$

u		dv
$2x$	+	$\cos 4x$
2	−	$\frac{1}{4}\sin 4x$
0		$-\frac{1}{16}\cos 4x$

$$\boxed{\frac{x}{2}\sin 4x + \frac{1}{8}\cos 4x + C}$$

[B]

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(7) $\int x^2 e^{2x} \, dx$

u		dv
x^2	+	e^{2x}
$2x$	−	$\frac{1}{2}e^{2x}$
2	+	$\frac{1}{4}e^{2x}$
0		$\frac{1}{8}e^{2x}$

$$\frac{x^2}{2}e^{2x} - \frac{x}{2}e^{2x} + \frac{1}{4}e^{2x} + c$$

$$\boxed{e^{2x}\left(\frac{x^2}{2} - \frac{x}{2} + \frac{1}{4}\right) + c}$$

[D]

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(8) $\int_1^e 2x \ln x \, dx$

u		dv
$\ln x$	+	$2x$
$\frac{1}{x}$	−	x^2

$2x \ln x = 0$

$x = 1$

$$x^2 \ln x \Big|_1^e - \int_1^e \frac{x^2}{x} \, dx$$

$$x^2 \ln x - \frac{x^2}{2} \Big|_1^e$$

$$e^2(1) - \frac{e^2}{2} - \left(0 - \frac{1}{2}\right)$$

$$\frac{2e^2}{2} - \frac{e^2}{2} + \frac{1}{2} = \boxed{\frac{e^2+1}{2}}$$

[A]

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(9) $\int_0^1 \left(\sqrt{2 \arctan x}\right)^2 dx = 2\int_0^1 \arctan x \, dx$

u	dv
$\arctan x$	dx
$\frac{1}{1+x^2}$	x

$$2\left[x \arctan x - \frac{1}{2}\int \frac{2x}{1+x^2} dx\right]\Big|_0^1$$

$$2\left[x \arctan x - \frac{1}{2}\ln|1+x^2|\right]\Big|_0^1$$

$$2\left[1\left(\frac{\pi}{4}\right) - \frac{1}{2}\ln 2 - \cancel{(0-0)}\right]$$

$$\boxed{\frac{\pi}{2} - \ln 2}$$

[E]

(10) $\displaystyle\int_0^2 x\, f''(x)\,dx$

u		dv
x	$+$	f''
1	$-$	f'
0		f

$$= x f'(x) - f(x) \Big|_0^2$$

$$2f'(2) - f(2) - (0 - f(0))$$

$$2(-1) - (-5) - (-3)$$

$$-2 + 5 + 3 = 6$$

$\boxed{C}$

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(11) $\displaystyle\int \frac{4x+2}{(x+3)(x+1)}\,dx \quad = \quad \frac{A}{x+3} + \frac{B}{x+1} = \frac{4x+2}{(x+3)(x+1)}$

$$A(x+1) + B(x+3) = 4x+2$$

$x = -1$ | $x = -3$
$2B = -2$ | $-2A = -10$
$B = -1$ | $A = 5$

$$= \int \frac{5}{x+3} - \frac{1}{x+1}\,dx$$

$\boxed{C}$

$$5\ln|x+3| - \ln|x+1| + C$$

$$\ln(x+3)^5 - \ln|x+1| + C = \boxed{\ln\left|\frac{(x+3)^5}{x+1}\right| + C}$$

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(12) $\displaystyle\int \frac{\sin x}{\cos x(\cos x - 1)}\,dx$

$u = \cos x$
$du = -\sin x\,dx$

$$\int \frac{-1}{u(u-1)}\,du$$

$$\frac{A}{u} + \frac{B}{u-1} = \frac{-1}{u(u-1)}$$

$$A(u-1) + Bu = -1$$

$u = 1$ | $u = 0$
$B = -1$ | $-A = -1$
 | $A = 1$

$$= \int \frac{1}{u} - \frac{1}{u-1}\,du$$

$$\ln|u| - \ln|u-1| + C$$

$$\ln\left|\frac{u}{u-1}\right| + C$$

$\boxed{E}$

$$\boxed{\ln\left|\frac{\cos x}{\cos x - 1}\right| + C}$$

(13) $\displaystyle\int \frac{x^3}{x^2-1}\,dx$ long divide first...

$$\begin{array}{r|l} & x + \frac{x}{x^2-1} \\ x^2-1 & x^3 \\ & -x^3+x \\ \hline & x \end{array}$$

$$\int x + \frac{x}{x^2-1}\,dx = \boxed{\frac{x^2}{2} + \frac{1}{2}\ln|x^2-1| + C}$$

None of the answers match....

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(14) $\displaystyle\int_2^\infty \frac{9}{x^2+x-2} = \lim_{b\to\infty}\int_2^b \frac{9}{(x+2)(x-1)}$

$$\frac{A}{x+2} + \frac{B}{x-1} = \frac{9}{(x+2)(x-1)}$$

$$A(x-1) + B(x+2) = 9$$

$x=1$: $3B = 9$, $B = 3$

$x=-2$: $-3A = 9$, $A = -3$

$$= \lim_{b\to\infty}\int_2^b \frac{3}{x-1} - \frac{3}{x+2}\,dx$$

$$= \lim_{b\to\infty} 3\ln|x-1| - 3\ln|x+2| \Big|_2^b$$

$$= \lim_{b\to\infty} \ln\left|\frac{(x-1)^3}{(x+2)^3}\right| \Big|_2^b$$

$$= \lim_{b\to\infty} \ln\frac{(b-1)^3}{(b+2)^3} - \ln\frac{1}{64}$$

$$\ln 1 - (\ln 1 - \ln 64)$$

$$0 - 0 + \ln 64 = \ln 4^3 = 3\ln 4$$

$$3\ln 2^2 = \boxed{6\ln 2}$$

$\boxed{D}$

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(15)

I $\displaystyle\int_1^\infty \frac{1}{x^{1/2}}\,dx = \lim_{b\to\infty}\int_1^b x^{-1/2}\,dx = \lim_{b\to\infty} 2x^{1/2}\Big|_1^b$

$$= \lim_{b\to\infty} 2\sqrt{b} - 2 = \infty \quad \boxed{\text{Diverges}}$$

II $\displaystyle\int_0^1 x^{-1/2}\,dx = \lim_{b\to 0^+} 2x^{1/2}\Big|_b^1$

$$= \lim_{b\to 0^+} 2 - 2\sqrt{b} = 2 \quad \boxed{\text{convergent}}$$

III $\displaystyle\int_1^\infty x^{-3/2}\,dx = \lim_{b\to\infty} -2x^{-1/2}\Big|_1^b = \lim_{b\to\infty} \underbrace{\cancel{\frac{-2}{\sqrt{b}}}}_{0} + \frac{2}{\sqrt{1}} = 2 \quad \boxed{\text{convergent}}$

$\boxed{D}$

(16) $\displaystyle\int_0^\infty x e^{-4x}\,dx = \lim_{b\to\infty}\int_0^b x e^{-4x}\,dx$

u		dv
x	$+$	e^{-4x}
1	$-$	$-\frac{1}{4}e^{-4x}$
0		$\frac{1}{16}e^{-4x}$

$$\lim_{b\to\infty}\left(-\frac{x}{4}e^{-4x} - \frac{1}{16}e^{-4x}\right|_0^b$$

$$\boxed{\lim_{b\to\infty}\left(\frac{-b}{4e^{4b}}\right.} - \frac{\cancel{1}}{16e^{4b}}\to 0 - 0 + \frac{1}{16}\Big)$$

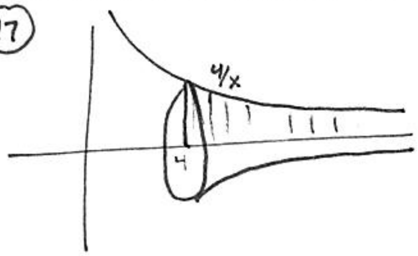
to truly show $\displaystyle\lim_{b\to\infty}\frac{-b}{4e^{4b}} = 0$, use L'Hopital's Rule

$$\lim_{b\to\infty}\frac{-1}{16e^{4b}} = 0$$

$$= \boxed{\frac{1}{16}}$$

$\boxed{B}$

(17)



$$V = \pi\int_4^\infty\left(\frac{4}{x}\right)^2 dx = 16\pi\lim_{b\to\infty}\int_4^b \frac{1}{x^2}\,dx \qquad \int x^{-2} = \frac{-1}{x}$$

$$= 16\pi\lim_{b\to\infty} \left.\frac{-1}{x}\right|_4^b$$

$$= 16\pi\lim_{b\to\infty} \underbrace{\frac{-1}{b}}_{\to 0} + \frac{1}{4}$$

$$= 16\pi\left(\frac{1}{4}\right) = \boxed{4\pi}$$

$\boxed{C}$

(18) $\displaystyle 2\lim_{b\to\infty}\int_1^b \frac{1}{2}\,\frac{1}{\sqrt{x}(x+1)}\,dx \qquad u = \sqrt{x} \quad du = \frac{1}{2\sqrt{x}}\,dx$

$$2\lim_{b\to\infty}\int_1^{\sqrt{b}}\frac{1}{1+u^2}\,du = 2\lim_{b\to\infty}\left.\arctan u\right|_1^{\sqrt{b}}$$

$$= 2\lim_{b\to\infty}\arctan\sqrt{b} - \arctan 1$$

$$= 2\left[\frac{\pi}{2} - \frac{\pi}{4}\right] = 2\left[\frac{\pi}{4}\right] = \boxed{\frac{\pi}{2}}$$

$\boxed{B}$

wicked cool!

(19) Asymptote @ $x = 1$

$$\lim_{b\to 1^-}\int_{-2}^b (x-1)^{-3}\,dx + \lim_{b\to 1^+}\int_b^4 (x-1)^{-3}\,dx$$

$$\lim_{b\to 1^-}\left.\frac{(x-1)^{-2}}{-2}\right|_{-2}^b$$

$$\lim_{b\to 1^-}\underbrace{\frac{-1}{2(b-1)^2}}_{\to\infty} + \frac{1}{2(9)}$$

Diverges