Semester I Fun Worksheet #2 Solutions☺

1. a.) $b = 2$
   b.) Yes, $f$ is differentiable when $b = 2$.

2. $a = -\frac{7}{4}, b = \frac{21}{4}, c = \frac{7}{2}$

3. a) $H'(1) = -12$ b) $H'(-2) = 21$ c) $H'(-2) = 48$

4. a) $x = 0, x = 2$ b) $x = 2, x = -5, x = -\frac{4}{5}$

5. The domain of $F$ is all real numbers so $F$ is continuous on the closed interval $[1,3]$.
   $F'(x) = 3x^2 - 10x - 3$. $F'$ exists for all real numbers so $F$ is differentiable on the open interval $(1,3)$.
   $c = \frac{7}{3}$

6. Discontinuous at $x = 0$ (Essential), $x = -1$ (Removable), and $x = 1$ (Removable)

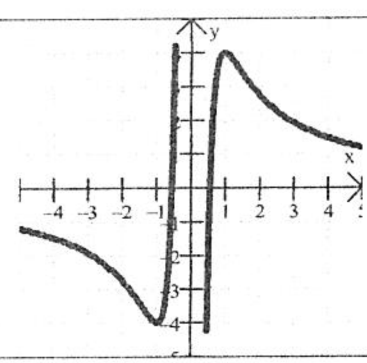
7. $\left.\frac{dz}{dt}\right|_{\substack{x=1\\y=2}} = -12$ units/second

8.

| | |
|---|---|
| a.) $v(0) = 32$ | b.) greatest height is 16 |
| c.) the time it takes to reach the ground is 2 | d.) the average velocity from $t = \frac{1}{2}$ to $t = \frac{3}{4}$ is 12 |

9. Moving right when $t \le -2$ or $t \ge \frac{1}{2}$ Moving left when $-2 \le t \le \frac{1}{2}$
   Velocity increasing when $t \ge -\frac{3}{4}$ Velocity decreasing when $t \le -\frac{3}{4}$
   Speeding up when $-2 < t < -\frac{3}{4}$ or $\frac{1}{2} < t < \infty$ Slowing down when $t < -2$ or $-\frac{3}{4} < t < \frac{1}{2}$

10.

| | | |
|---|---|---|
| a.) $\left(-\frac{1}{\sqrt{3}}, 0\right)$ and $\left(\frac{1}{\sqrt{3}}, 0\right)$ | b.) $y = 0$ | c.) $x = 0$ |
| d.) $f'(x) = \frac{-6x^2 + 6}{x^4}$ | e.) $f''(x) = \frac{12x^2 - 24}{x^5}$ | f.) $(-1, -4)$ relative minimum<br>$(1, 4)$ relative maximum |
| g.) increasing on $[-1, 0) \cup (0, 1]$<br>decreasing on $(-\infty, -1] \cup [1, \infty)$ | h.) $\left(\sqrt{2}, \frac{5\sqrt{2}}{2}\right)$ $\left(-\sqrt{2}, -\frac{5\sqrt{2}}{2}\right)$ | i.) $(-\infty, -\sqrt{2}) \cup (0, \sqrt{2})$ |
| j.) $\lim_{x \to 0^+} f(x) = -\infty$ $\lim_{x \to 0^-} f(x) = \infty$<br>$\lim_{x \to +\infty} f(x) = 0$ $\lim_{x \to -\infty} f(x) = 0$ | | k.)  |

11. E
12. C
13. B
14. E

15. Minimum value $= -\frac{1}{3}$ Maximum value $= \frac{\sqrt{2}}{4}$

16. The domain of $F$ is all real numbers so $F$ is continuous on $[0,2]$. $F'(x)=3x^2+8x-12$ and it exists for all real numbers, so $F$ is differentiable on $(0,2)$. $F(0)=0$ and $F(2)=0$. Thus all of the hypotheses of Rolle's Theorem have been satisfied. $c=\frac{-4+2\sqrt{13}}{3}$

17.

a.) $[3,6]$ $f'(x)>0$ on $(3,6)$

b.) $x=6$ $f'$ changes from positive to negative at $x=6$

c.) $x=1, x=2, x=4.2, x=8$

d.) $(1,2)\cup(4.2,8)$ because $f'(x)$ is decreasing on these intervals

e.) $x=0$ because $f$ is decreasing on $[0,2]$ so $f(2)<f(0)$

f.) $x=3$ because $f$ is increasing on $[3,6]$ so $f(6)>f(3)$

g.) $y-24=-2(x-2)$

h.) $f(2.03)\approx 23.94$

i.) The tangent line yields a smaller value than $f(2.03)$. At $x=2.03$, $f$ is concave up, so the tangent line lies below $f$, thus yielding underestimates.

18. a.) $\frac{dy}{dx}=\frac{2x}{3y^2+2y-5}$

b.) Horizontal tangents when $x=0$
Vertical tangents when $y=1$ or $y=-\frac{5}{3}$

19. a.) $\frac{dP}{dt}=\frac{24}{\pi}$ in/sec

b.) Let $A=A_{square}-A_{circle}$

$$\left.\frac{dA}{dt}\right|_{A_{circle}=25\pi}=\frac{120}{\pi}-30 \text{ in}^2\text{/sec}$$