# Double Trouble

① $y = \sqrt{x^2+1} \cdot \sin(2x)$ use Product Rule f·g

$(x^2+1)^{1/2} \cos(2x) \cdot 2 + \sin(2x) \frac{1}{2}(x^2+1)^{-1/2} \cdot 2x$

$(x^2+1)^{-1/2}\left[(x^2+1) \cdot 2\cos 2x + x\sin(2x)\right]$

$$\boxed{\frac{2(x^2+1)\cos 2x + x\sin(2x)}{\sqrt{x^2+1}}}$$

double ang.

② $f(x) = \sin^2(\pi x)$ use chain Rule $u^2$

$= [\sin(\pi x)]^2$

$f'(x) = 2\sin(\pi x) \cdot \cos \pi x \cdot \pi$

$= \boxed{2\pi \sin(\pi x) \cdot \cos(\pi x)}$ also by double angle identity $\sin 2u = 2\sin u \cos u$

$\boxed{\pi \sin(2\pi x)}$

③ $g(x) = \cos^4(\sqrt{\pi x})$ use chain Rule $u^4$

$4u^3 \cdot du$

$= [\cos\sqrt{\pi x}]^4$

$g'(x) = 4[\cos\sqrt{\pi x}]^3 \cdot -\sin\sqrt{\pi x} \cdot \frac{1}{2}(\pi x)^{-1/2} \cdot \pi$ whoa fxn within a fxn within a fxn!

$$= \boxed{\frac{-2\pi \cos^3\sqrt{\pi x} \sin\sqrt{\pi x}}{\sqrt{\pi x}}}$$

also with $2\sin u \cos u = \sin 2u$

$$\frac{-\pi(\sin 2\sqrt{\pi x})\cos^2\sqrt{\pi x}}{\sqrt{\pi x}}$$

④ $h(x) = \dfrac{\sqrt{x^4+4}}{\cot x}$ Quotient Rule

$$h'(x) = \frac{\cot x \cdot \frac{1}{2}(x^4+4)^{-1/2}(4x^3) - (x^4+4)^{1/2}(-\csc^2 x)}{\cot^2 x}$$

$$= \frac{(x^4+4)^{-1/2}\left[2x^3\cot x + (x^4+4)\csc^2 x\right]}{\cot^2 x}$$

$$= \boxed{\frac{2x^3\cot x + (x^4+4)\csc^2 x}{\sqrt{x^4+4}\,\cot^2 x}} \quad \tan^2 x \quad = \boxed{\frac{2x^3\tan x + (x^4+4)\sec^2 x}{\sqrt{x^4+4}}}$$

(5) $y = \dfrac{x^3+1}{\sec(2\pi x)}$ quotient Rule

$$y' = \frac{\sec(2\pi x)(3x^2) - (x^3+1)\sec(2\pi x)\tan(2\pi x)\cdot 2\pi}{\sec^2(2\pi x)}$$

$$= \frac{\cancel{\sec(2\pi x)}\left[3x^2 - (x^3+1)\tan(2\pi x)(2\pi)\right]}{\sec^{\cancel{2}}(2\pi x) \quad \sec(2\pi x)}$$

$$= \left[3x^2 - (x^3+1)\tan(2\pi x)\cdot 2\pi\right]\cos(2\pi x)$$

$$= \boxed{3x^2\cos(2\pi x) - 2\pi(x^3+1)\sin(2\pi x)}$$

(6) $h(x) = \dfrac{f(20x)}{g(11x)}$ quotient

$$h'(x) = \frac{g(11x)\, f'(20x)\cdot 20 - f(20x)\, g'(11x)\cdot 11}{[g(11x)]^2}$$

$$= \boxed{\frac{20\, g(11x)\, f'(20x) - 11\, f(20x)\, g'(11x)}{[g(11x)]^2}}$$

(7) $f(t) = g(5t)h(2t) - h(5t)g(2t) + g^2(t)h^3(t)$

$$f'(t) = g(5t)\,h'(2t)\cdot 2 + h(2t)\,g'(5t)\cdot 5 - h(5t)\,g'(2t)\cdot 2 - g(2t)\,h'(5t)\cdot 5 + g^2(t)\,3h^2(t)\,h'(t) + h^3(t)\,2g(t)\,g'(t)$$

Bonus

(8) $\cos(xy) = xy$

$$\sin(xy)\left(x\frac{dy}{dx} + y\right) = x\frac{dy}{dx} + y$$

$$x\frac{dy}{dx}\sin(xy) + y\sin(xy) = x\frac{dy}{dx} + y$$

$$x\frac{dy}{dx}\sin(xy) - x\frac{dy}{dx} = y - y\sin(xy)$$

$$\frac{dy}{dx}\left[x\sin(xy) - x\right] = y - y\sin(xy)$$

$$\frac{dy}{dx} = \frac{y - y\sin(xy)}{x\sin(xy) - x}$$