# Implicit Differentiation

Note Title 10/19/2011

$$\frac{d}{dx} \quad 2x + 3y = 7$$

$$3y = -2x + 7$$

$$y = -\frac{2}{3}x + \frac{7}{3}$$

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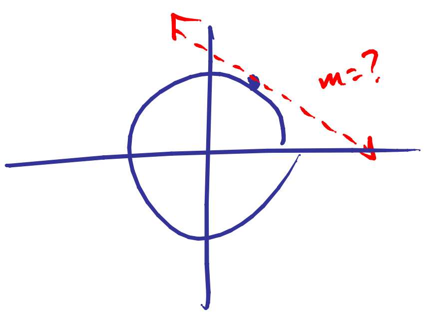
$$x^2 + y^2 = 25$$

slope of tang line at $(3,4)$




$$x^2 + y^2 = 25$$

$$y = \pm\sqrt{25 - x^2}$$ explicit

$$y = (25 - x^2)^{1/2}$$

$\square^{1/2}$ $\quad \sqrt{25-x^2}$

$$y' = \frac{1}{2}(25 - x^2)^{-1/2}(-2x)$$

$\frac{1}{2}\square^{-1/2}(-2x)$

$$\boxed{y' = \frac{-x}{\sqrt{25 - x^2}}}$$

$$\boxed{m = \frac{-3}{4}}$$

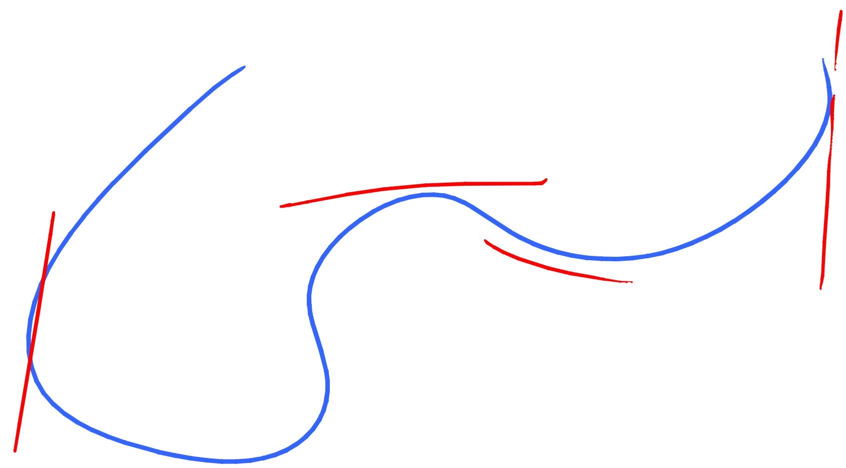
$$\boxed{x+y = \sin y}$$ Implicit Relation

$\arcsin(x+y) = y$

$y = \underline{f(x)}$ ?

$2x + 3y = 7$ implicit

$y = -\frac{2}{3}x + \frac{7}{3}$ explicit



$\boxed{\frac{d}{dx}}$ = "The Derivative of $\square$ with respect to x"

$\frac{d}{dt}$ $\frac{d}{dr}$ $A = \pi r^2$

$\frac{d}{d\boxed{x}} 2\boxed{x} = 2$

$\frac{d}{d\boxed{\text{horse}}} \cot\boxed{\text{horse}}$

$-\csc^2(\text{horse})$

$\frac{d}{d\boxed{t}} \sin\boxed{t} = \cos t$

$\frac{d}{dx}\ 2x$ chain Rule?

$2\,\square \qquad 2\,\frac{d}{dx}\boxed{x} \qquad \frac{dx}{dx}$

$\frac{d}{dx}\ y = 1$

$y = f(x)$

$\frac{d}{dx}\ f(x) = f'(x) \cdot \frac{dx}{dx}$ $'$

$y'$

$\frac{d}{dx}\ y^2 = 2y \cdot y'$

$\frac{d}{dx}\ \square^2 = 2\,\square\ d\,\square$

$\frac{d}{dx}\ x^2 + y^2 = \frac{d}{dx}25$ $\qquad y' \qquad \frac{d}{dx}\ x =$

$\frac{d}{dx}\ x^2 + \frac{d}{dx}\ y^2 = \frac{d}{dx}\ 25$

$2x + 2y\ y' = 0$

$\frac{d}{dy}\ x = \sqrt{25-y^2}$

$2y\ y' = -2x$

$\boxed{y' = \frac{-x}{y}}$ $\qquad \left(\frac{-3}{4}\right)$

$$y' = \frac{-x}{\boxed{\sqrt{25-x^2}}}$$

Solved for $y = \pm\sqrt{25-x^2}$

$$\frac{d}{dx}\, x^2 + 3y + y^2 = \frac{d}{dx}\, 2x$$

Find $\boxed{\frac{dy}{dx}} = y'$

interchangeable

$$2x + 3\frac{dy}{dx} + 2y\frac{dy}{dx} = 2$$

$$3\frac{dy}{dx} + 2y\frac{dy}{dx} = 2 - 2x$$

Factor $\frac{dy}{dx}(3+2y) = 2-2x$

Divide $\frac{dy}{dx} = \frac{2-2x}{3+2y}$

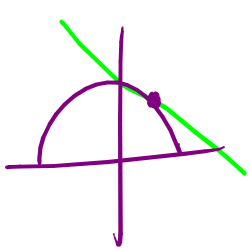
$$xy + \sin x = 4 \qquad \frac{dy}{dx}\,?$$

$$x^2 + y^2 = 25$$

$$y^2 = 25 - x^2$$

$(3,4)$

$$y = \pm\sqrt{25-x^2}$$



$$y = (25-x^2)^{1/2}$$

$$y' = \cancel{\tfrac{1}{2}}(25-x^2)^{-1/2}(-\cancel{2}x)$$

$$y' = \frac{-x}{\sqrt{25-x^2}}$$

$y'(3) = \boxed{\frac{-3}{4}}$

$x^2 + y^2 = 25$ implicitly

$y = \pm\sqrt{25-x^2}$ explicitly

$\boxed{\frac{d}{dt}} 3t^2 + 10t = 6t + 10$

$3\square^2 + 10\square$

$6\boxed{t}\frac{d}{dt}\boxed{t} + 10\frac{d}{dt}\boxed{t}$

$6t \quad + 10$

$\frac{d}{dt} r$ $\quad$ $\boxed{\frac{d}{dx} y}$ $\quad$ $\frac{d}{dz} 4z + 2y$

$y = f(x)$

$\frac{d}{dx} y = \frac{d}{dx} f(x)$ chainrule

$f'(x) \cdot \frac{dx}{dx}$

$f'(x)$

$y' = \frac{dy}{dx}$

$\frac{d}{dx} x^2 + y^2 = \frac{d}{dx} 25$

Find $\boxed{\frac{dy}{dx}}$ at $(3,4)$

$\square^2$

$2\square\, d\square$

$2x + \boxed{2y \cdot \frac{dy}{dx}} = 0$

$2y \frac{dy}{dx} = -2x$

$\frac{dy}{dx} = \frac{-\cancel{2}x}{\cancel{2}y} = \boxed{\frac{-x}{y}}$

$\boxed{\frac{-x}{\sqrt{25-x^2}}}$

$\frac{d}{dt} \cancel{x}^2 + 3x$

$2x \frac{dx}{dt} + 3 \frac{dx}{dt}$

$\frac{dy}{dx} = y'$

interchangable

$2x\, x' + 3x'$

$\frac{dx}{dt} = x'$

$x'(2x+3)$

Why implicit Differentiation?

$\frac{d}{dx} x^2 + xy + y^3 = \frac{d}{dx} 10$

$\square^3$

To find $\frac{dy}{dx}$

1. $\frac{d}{dx}$ both sides
2. Move all $y'$ to one side
3. Factor $y'$
4. Divide

$2x + xy' + y + 3y^2 \cdot y' = 0$

$xy' + 3y^2y' = -2x - y$

$y'(x + 3y^2) = -2x - y$

$$\boxed{y' = \frac{-2x - y}{x + 3y^2}}$$

$\boxed{\frac{d}{dx}} \rightarrow$

$\frac{d}{dx} 2x = 2$

$2\square = 2 \frac{d}{dx}\boxed{x}$

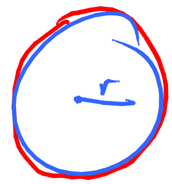
$\frac{d}{dx} x^2 \qquad 2x$

$\square^2 \qquad 2\square \frac{d}{dx}\boxed{x}$

$\frac{d}{dx} \sin x \qquad \cos x$

$\sin\square \qquad \cos\square \frac{d}{dx}\boxed{x}$

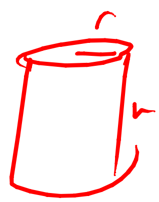
$$\boxed{\frac{d}{dr}} \overset{A}{\pi r^2} = 2\pi r$$



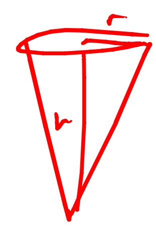

$$\frac{d}{dr}\frac{4}{3}\pi r^3 = \overset{SA}{4\pi r^2}$$

$$\frac{d}{dx} f(x) = f'(x).$$

$$\frac{d}{dr}\pi r^2 h$$




$\frac{1}{3}\pi r^2 h$




roc of volume

Think chain Rule

$$\frac{d}{dx} y =$$

$$\frac{d}{dx}\square = \frac{d}{dx}\square$$

$$\frac{d}{dx} y = \frac{dy}{dx} \qquad y = f(x)$$

$$\frac{d}{dx} y^2 =$$

$$\boxed{y}^2 = 2\boxed{y}\cdot\frac{d}{dx}\boxed{y} = 2y\frac{dy}{dx}$$

$$\frac{d}{dx}\cos y =$$

$$\cos \square = -\sin \square \cdot \frac{d}{dx}\square$$

$$-\sin y \cdot \frac{dy}{dx}$$

$$y' = \frac{dy}{dx}$$

$$\frac{d}{dx} x y = x \cdot y' + y(1) = xy' + y$$

$$\frac{d}{dx} x^2 + y^2 = \frac{d}{dx} 25 \qquad \text{tan } (3,4)$$

Imp Diff

1. $\frac{d}{dx}$ Both sides
2. $\frac{dy}{dx}$ on one side

$$2x + 2y \cdot y' = 0$$

$$2yy' = -2x$$

$$y' = \frac{-\cancel{2}x}{\cancel{2}y} = \frac{-x}{y}$$

$$\frac{dy}{dx} = \frac{-x}{y} \qquad \frac{-x}{\sqrt{25-x^2}}$$

$$\frac{d}{dx} x^2 + xy + y^3 = \frac{d}{dx} 25$$

m tan $(5,0)$ m norm

eqn of tan line

$$2x + xy' + y + 3y^2y' = 0$$

$$xy' + 3y^2y' = -2x - y$$

$$y'(x + 3y^2) = -2x - y$$

$$y' = \frac{-2x - y}{x + 3y^2}$$

$$y'(5,0) = \frac{-2(5) - \cancel{0}}{5 + \cancel{3}(0)^2} = \boxed{-2} \qquad (5,0)$$

$$y - 0 = -2(x-5)$$

$$y = -2(x-5)$$

$$-2x + 10$$

Normal $\frac{1}{2}$

$$y = \frac{1}{2}(x-5)$$

$$\sin^2 y + \cos^2 x = y^2$$

$$\frac{d}{dx}[\sin y]^2 + [\cos x]^2 = \frac{d}{dx} y^2$$

$$[illegible] + 2\cos x(-\sin x) = 2y\,y'$$

$$2\sin y\cos y\,y' - 2y\,y' = 2\sin x\cos x$$

$$y'(2\sin y\cos y - 2y) = 2\sin x\cos x$$

$$y' = \frac{\sin x\cos x}{\sin y\cos y - y}$$