Integration Techniques

1. $\int \frac{6}{x^2-1} dx$

2. $\int \frac{6}{x^2+1} dx$

3. $\int x e^{x^2} dx$

4. $\int t\sqrt{t^2+2} dt$

5. $\int x^2 \sin(2x) dx$

6. $\int \frac{\ln x^2}{x} dx$

7. $\int \frac{x^2+2x+4}{x-3} dx$

8. $\int_1^{\infty} e^{-2x} dx$

9. $\int_0^{\frac{\pi}{2}} \tan x dx$

10. $\int 4 \arccos x dx$

Int. Techniques

① $\int \frac{6}{x^2-1}\,dx$ $\qquad \frac{6}{x^2-1} = \frac{6}{(x-1)(x+1)} = \frac{A}{x-1} + \frac{B}{x+1}$

$6 = A(x+1) + B(x-1)$

$x=-1$ $\qquad$ $x=1$

$6 = -2B$ $\qquad$ $3 = A$

$\boxed{-3 = B}$

$= \int \frac{3}{x-1} - \frac{3}{x+1}\,dx$

$= \boxed{3\ln|x-1| - 3\ln|x+1| + C}$

or $= \ln\left|\frac{x-1}{x+1}\right|^3 + C$

② $\int \frac{6}{x^2+1}\,dx = 6\int \frac{1}{1+x^2}\,dx = \boxed{6\arctan x + C}$

③ $\int x e^{x^2}\,dx$ $\qquad u = x^2$, $du = 2x\,dx$

$\frac{1}{2}\int 2x\, e^{x^2}\,dx =$

$\frac{1}{2}\int e^u\,du = \frac{1}{2}e^u + C \Rightarrow \boxed{\frac{1}{2}e^{x^2} + C}$

④ $\int t\,(t^2+2)^{1/2}\,dt$ $\qquad u = t^2+2$, $du = 2t\,dt$

$\frac{1}{2}\int 2t\,(t^2+2)^{1/2}\,dt$

$\frac{2}{3}\frac{1}{2}(t^2+2)^{3/2} + C = \boxed{\frac{1}{3}\sqrt{(t^2+2)^3} + C}$

⑤ $\int x^2 \sin(2x)\,dx$

u		dv
x^2	$+$ ↘	$\sin 2x$
$2x$	$-$ ↘	$-\frac{1}{2}\cos 2x$
2	$+$ ↘	$-\frac{1}{4}\sin 2x$
0		$\frac{1}{8}\cos 2x$

$\boxed{-\frac{1}{2}x^2\cos 2x + \frac{1}{2}2x\sin 2x + \frac{1}{4}\cos 2x + C}$

⑥ $\int \frac{\ln x^2}{x}\,dx = \int \frac{2\ln x}{x}\,dx = 2\int \frac{\ln x}{x}\,dx$ $\quad u=\ln x,\ du=\frac{1}{x}dx$

$= 2\frac{(\ln x)^2}{2} + C = \boxed{(\ln x)^2 + C}$

⑦ $\int \frac{x^2+2x+4}{x-3}\,dx = \int x+5+\frac{19}{x-3}\,dx = \boxed{\frac{x^2}{2} + 5x + 19\ln|x-3| + C}$

$$\begin{array}{r} x+5+\frac{19}{x-3} \\ x-3\,\overline{)\,x^2+2x+4} \\ -x^2+3x \\ \hline 5x+4 \\ -5x+15 \\ \hline 19 \end{array}$$

⑧ $\int_1^\infty e^{-2x}\,dx = \lim_{b\to\infty} -\frac{1}{2}\int_1^b -2e^{-2x}\,dx = -\frac{1}{2}\lim_{b\to\infty} e^{-2x}\Big|_1^b$

$= -\frac{1}{2}\lim_{b\to\infty} \frac{1}{e^{2b}} - \frac{1}{e^2}$ $\quad \frac{1}{e^{2b}} \to 0$

$= \boxed{\frac{1}{2e^2}}$ converges

⑨ $\int_0^{\pi/2} \tan x\,dx = -\ln|\cos x|\Big|_0^{\pi/2} = -(\ln|\cos \pi/2| - \ln|\cos$

$= \lim_{b\to \pi/2^-} \int_0^b \tan x\,dx = -\lim_{b\to\pi/2^-} \ln|\cos x|\Big|_0^b$

$= -\lim_{b\to\pi/2^-} \ln|\cos b| - \ln|\cos 0|$

$= -(-\infty - 0) = \infty$ $\boxed{\text{diverges}}$

⑩ $\int 4\arccos x\,dx$

u	dv
$\arccos x$	dx
$\frac{-1}{\sqrt{1-x^2}}$	x

$= 4\left(x\arccos x - \int \frac{-x}{\sqrt{1-x^2}}\,dx\right)$

$= 4\left(x\arccos x + \frac{-1}{2}\int \frac{-2x}{\sqrt{1-x^2}}\,dx\right)$ $\quad u=1-x^2,\ du=-2x\,dx$

$= 4\left(x\arccos x - \frac{1}{2}\sqrt{1-x^2}\right) + C$

$= \boxed{4x\arccos x - 4\sqrt{1-x^2} + C}$