11/10/10

Techniques of integration Review sheet

① $\int \frac{e^x}{e^x+1}\,dx$ $u = e^x+1$, $du = e^x dx$

$\int \frac{du}{u} = \ln|u| + C$

$\boxed{\ln|e^x+1| + C}$

② $\int \frac{\cos x}{\sin^2 x}\,dx$ $u = \sin x$, $du = \cos x\,dx$

$\int \frac{1}{u^2}\,du = \int u^{-2}\,du$

$= -u^{-1} + C$

$= \frac{-1}{\sin x} + C$

$\boxed{= -\csc x + C}$

or ... recognize dint. rule

$\int \frac{\cos}{\sin}\cdot\frac{1}{\sin}\,dx = \int \cot x \csc x\,dx =$

$\boxed{-\csc x + C}$

③ $\int x^3 \cos x\,dx$

$$\begin{array}{ccc} u & & dv \\ x^3 & + & \cos x \\ 3x^2 & - & \sin x \\ 6x & + & -\cos x \\ 6 & - & -\sin x \\ 0 & & \cos x \end{array}$$

$\boxed{x^3\sin x + 3x^2\cos x - 6x\sin x - 6\cos x + C}$

④ $\int \frac{\ln x}{x^2}\,dx =$

$$\begin{array}{ccc} u & & dv \\ \cancel{\ln x} & \cancel{+} & \cancel{x^2} \\ \cancel{\frac{1}{x}} & \cancel{-\int} & \cancel{\frac{1}{3}x^3} \end{array}$$

$\cancel{\frac{1}{3}x^3\ln x - \int \frac{1}{x}\left(\frac{1}{3}x^3\right)dx}$

$\cancel{\frac{1}{3}x^3\ln x - \frac{1}{3}\int x^2\,dx}$

$\cancel{\frac{1}{3}x^3\ln x - \frac{1}{3}\frac{1}{3}x^3 + C}$

$\cancel{\boxed{\frac{1}{3}x^3\ln x - \frac{1}{9}x^3 + C}}$

Duh purvis!

$$\begin{array}{ccc} u & & dv \\ \ln x & + & x^{-2} \\ \frac{1}{x} & -\int & -x^{-1} \end{array}$$

$-\frac{\ln x}{x} - \int -\frac{1}{x^2}\,dx$

$-\frac{\ln x}{x} + \int x^{-2}\,dx$

$-\frac{\ln x}{x} + -x^{-1} + C$

$\boxed{\frac{-\ln x - 1}{x} + C}$

⑤ $\int \frac{1}{x^2-4x+3}\,dx = \int \frac{1}{(x-3)(x-1)}\,dx$

$\frac{A}{x-3} + \frac{B}{x-1} = \frac{1}{(x-3)(x-1)}$

$A(x-1) + B(x-3) = 1$

let $x=1$: $-2B = 1$, $B = -\frac{1}{2}$

let $x=3$: $2A = 1$, $A = \frac{1}{2}$

$= \int \frac{\frac{1}{2}}{x-3} + \frac{-\frac{1}{2}}{x-1}\,dx$

$\boxed{= \frac{1}{2}\ln|x-3| - \frac{1}{2}\ln|x-1| + C}$

⑥ $\int \frac{x-3}{x(x+1)(x+3)}\,dx$

$$\frac{A}{x} + \frac{B}{x+1} + \frac{C}{x+3} = \frac{x-3}{x(x+1)(x+3)}$$

$$A(x+1)(x+3) + B(x)(x+3) + C\,x(x+1) = x-3$$

let $x=-1$: $-2B = -4$, $\underline{B = 2}$

let $x = 0$: $3A = -3$, $\underline{A = -1}$

let $x = -3$: $6C = -6$, $\underline{C = -1}$

$$\int \frac{-1}{x} + \frac{2}{x+1} + \frac{-1}{x+3}\,dx$$

$$\boxed{-\ln|x| + 2\ln|x+1| - \ln|x+3| + C}$$

⑦ $\int \frac{2x^3}{x^2-x-6}\,dx = \int 2x+2 + \frac{14x+12}{x^2-x-6}\,dx$

$$x^2-x-6 \overline{)\,2x^3} \quad\to\quad 2x+2 + \frac{14x+12}{x^2-x-6}$$

$-2x^3+2x^2+12x$

$2x^2+12x$

$-2x^2+2x+12$

$14x+12$

$$\int 2x+2+\frac{\frac{64}{5}}{x-3} + \frac{\frac{16}{5}}{x+2}\,dx$$

$$\boxed{x^2+2x+\frac{54}{5}\ln|x-3| + \frac{16}{5}\ln|x+2| + C}$$

$$\frac{14x+12}{x^2-x-6} = \frac{A}{x-3} + \frac{B}{x+2}$$

$$14x+12 = A(x+2) + B(x-3)$$

Let $x=-2$: $-16 = -5B$, $\frac{16}{5} = B$

let $x=3$: $54 = 5A$, $\frac{54}{5} = A$

⑧ $\int(\cos^3 x)(\sin^2 x)\,dx$

$\int \cos x \cdot \cos^2 x \cdot \sin^2 x\,dx$

$\int \cos x\,(1-\sin^2 x)\cdot \sin^2 x\,dx$

$\int(\sin^2 - \sin^4 x)\cos x\,dx$

$\int \sin^2 x\cos x - \sin^4 x\cos x\,dx$

$$\int \sin^2 x\cos x\,dx - \int \sin^4 x\cos x\,dx$$

$u=\sin x$, $du=\cos x\,dx$ $\qquad$ $u=\sin^4 x$, $du=\cos x$

$$\int u^2\,du - \int u^4\,du$$

$$\frac{1}{3}u^3 - \frac{u^5}{5} + c$$

$$\boxed{\frac{1}{3}\sin^3 x - \frac{1}{5}\sin^5 x + C}$$

(9) $$\int 2\cos^3 x\,dx$$

$$2\int \cos^2 \cdot \cos x\,dx$$

$$2\int (1-\sin^2 x)\cos x\,dx$$

$$2\int \cos x - \sin^2 \cancel{x}\cos x\,dx$$

$$2\left[\int \cos x\,dx - \int \sin^2 x\cos x\,dx\right]$$ $u = \sin x$, $du = \cos x\,dx$

$$2\left[\sin x - \frac{1}{3}\sin^3 x\right] + c$$

$$\boxed{2\sin x - \frac{2}{3}\sin^3 x + C}$$

(10) $$\int_0^1 x f''(x) = x f'(x) - f(x)\Big|_0^1$$

u		dv
x	+	f''
1	−	f'
0		f

$$1f'(1) - f(1) - (0f'(0) - f(0))$$

$$1(2) - 5 - (0-6)$$

$$-3 + 6 = \boxed{3}$$

(11) $$\int x^3 e^{x^2}\,dx$$ $U = x^2$, $du = 2x\,dx$

$$\int x \cdot e^{\cancel{x}^2} \cdot x^2\,dx$$

$$\frac{1}{2}\int 2x \cdot e^u \cdot u\,dx$$

$$\frac{1}{2}\int u e^u\,du$$

u		dv
u	+	e^u
1	−	e^u
0		e^u

$$\frac{1}{2}\left(ue^u - e^u\right) + c$$

$$\boxed{\frac{1}{2}x^2e^{x^2} - \frac{1}{2}e^{x^2} + C}$$

(12) $\int \cos^4 x \sin^2 x \, dx$

$$\boxed{\frac{1}{12}x - \frac{\sin 4x}{48} + \frac{\sin^3 2x}{36} + C}$$ or so it says...

(13) $\int_1^2 x f'(x^2)\, dx$

$u = x^2$
$du = 2x\,dx$

$\frac{1}{2}\int_1^2 f'(u)\ 2x\,dx$

$\frac{1}{2}\int_1^4 f'(u)\ du$

$\frac{1}{2} f(u)\Big|_1^4$

$\frac{1}{2}\left(f(4) - f(1)\right) = \frac{1}{2}(-7 - 2) = \boxed{-9/2}$

4) $\int_1^3 x f''(x)\, dx$

u		dv
x	+	f''
1	−	f'
0		f

$x f'(x) - f(x)\Big|_1^3 = 3f'(3) - f(3) - (1f'(1) - f(1))$

$3\cdot(2) - (-3) - (-1 - 2)$

$6 + 3 + 3 = \boxed{12}$