MVT &
Rolle's
Theorem

03

3. The conditions necessary to apply Rolle's Theorem are: 1) f is continuous on $[a, b]$; 2) f is differentiable on (a, b); and 3) $f(a) = f(b)$.

Which one of the following functions does NOT satisfy at least one of the above conditions?

(A) $f(x) = x^3 - x$ on $[0, 1]$
(B) $f(x) = \sqrt{8 - x^3}$ on $[-2, 2]$
(C) $f(x) = x^{\frac{4}{3}} - 1$ on $[-1, 1]$
(D) $f(x) = \dfrac{x^2 - 4}{x - 3}$ on $[-2, 2]$
(E) $f(x) = x^2 - 4x$ on $[0, 4]$

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21. Find the value of c which satisfies Rolle's Theorem for the function $f(x) = \sin\left(x^2\right)$ on $\left[0, \sqrt{\pi}\right]$.

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12. If $f'(x)$ exists for all x and $f(1) = 10$ and $f(8) = -4$, then, for at least one value of c in the open interval (1, 8), which of the following must be true?

(A) $f(c) = 12$
(B) $f(c) = -12$
(C) $f'(c) = 2$
(D) $f'(c) = -2$
(E) $f(c^2) = 16$

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21. Find a positive value of x that satisfies the Mean Value Theorem for $f(x) = \sin x$ on the closed interval $\left[-\dfrac{3\pi}{2}, \dfrac{3\pi}{2}\right]$.

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12. The function $f(x) = 4\sqrt[3]{x^2} - 4$ does not satisfy Rolle's Theorem on the interval $[-1, 1]$ because

(A) $f(0) \neq 0$
(B) $f(1) = f(-1)$
(C) $f(x)$ is not continuous on $[-1, 1]$
(D) $f(2) > f(1)$
(E) $f(x)$ is not differentiable on $(1, -1)$

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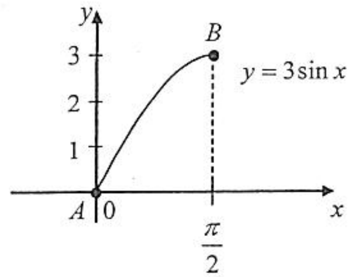
19.



The x-coordinate of the point on the curve $y = 3\sin x$, between points A and B, where the tangent to the curve is parallel to the secant line joining points A and B is approximately

(A) 0.591 (B) 0.623 (C) 0.750 (D) 0.796 (E) 0.881

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11. The function $f(x) = x^{\frac{2}{3}} - 1$ does not satisfy Rolle's Theorem on the interval $[-1, 1]$ because

(A) $f(-1) \neq 0$
(B) $f(x)$ is not differentiable on $(1, -1)$
(C) $f(x)$ is not continuous on $[-1, 1]$
(D) $f(1) > f(-1)$
(E) $f(0) < 0$

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27. The value of c which satisfies Rolle's Theorem for the function $f(x) = \sin\left(x^2\right)$ on $\left[0, \sqrt{\pi}\right]$ is

(A) $\sqrt{\frac{\pi}{6}}$ (B) $\sqrt{\frac{\pi}{4}}$ (C) $\sqrt{\frac{\pi}{3}}$ (D) $\sqrt{\frac{\pi}{2}}$
(E) none of the above

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15. The Mean Value Theorem states that if $y = f(x)$ is continuous on the closed interval $[a, b]$ and differentiable on the open interval (a, b), then there exists at least one point c in (a, b) such that $f'(c) = \frac{f(b) - f(a)}{b - a}$. If $f(x) = \begin{cases} 2x - x^2, & \text{if } x < 1 \\ 2x^2 - 4x + 3, & \text{if } x \geq 1 \end{cases}$, find two values of c in the interval $[a, b]$, where $a = 0$, $b = 2$, that satisfy the Mean Value Theorem.

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27. Which of the following does NOT satisfy the conditions for Rolle's Theorem?

(A) $f(x) = x^{\frac{2}{3}} - 1$ on $[-1, 1]$ (B) $f(x) = \sqrt{4 - x^2}$ on $[-2, 2]$

(C) $f(x) = \sin x$ on $[0, \pi]$ (D) $f(x) = x^4 - 1$ on $[-1, 1]$

(E) all of the above satisfy the conditions

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13. Find the point on the curve $y = x^2$ where the tangent to the curve is parallel to the secant line connecting $(-1, 1)$ and $(2, 4)$.

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14. If $f(x) = \sqrt[3]{x}$, $a = 0$, $b = 8$, and $f(b) - f(a) = (b - a)f'(c)$, then the value of c is

(A) $\frac{4}{3}\sqrt{2}$ (B) $\frac{9}{8}\sqrt{2}$ (C) $\frac{8}{9}\sqrt{2}$ (D) $\frac{4}{3}\sqrt{3}$ (E) $\frac{8}{9}\sqrt{3}$

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14. There are two values for c which satisfy the Mean Value Theorem, $f'(c) = \dfrac{f(b) - f(a)}{b - a}$, for the function $f(x) = (x - 1)^3$ from $x = -1$ to $x = 3$. Find these values.

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9. The point $(c, f(c))$ on the curve $f(x) = \sqrt{x}$ between $x = a = 0$ and $x = b = 4$ that satisfies $f'(c) = \dfrac{f(b) - f(a)}{b - a}$ is

(A) $\left(\frac{1}{2}, \frac{\sqrt{2}}{2}\right)$ (B) $(1, 1)$ (C) $(2, \sqrt{2})$ (D) $(3, \sqrt{3})$

(E) none of the above

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9. The value of c on $[0, 1]$ that satisfies Rolle's Theorem for $f(x) = x - x^{\frac{1}{3}}$, is

(A) $\frac{1}{27}$ (B) $\frac{\sqrt{3}}{9}$ (C) $\frac{1}{3}$ (D) $\frac{\sqrt{3}}{3}$ (E) $\frac{2}{3}\sqrt{3}$

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14. If $f(x) = 2x^3$, $a = 0$, $b = 3$, and $f(b) - f(a) = (b - a)f'(c)$, then the value of c in the interval $[0, 3]$ is

(A) $\sqrt{2}$ (B) $\sqrt{3}$ (C) 2 (D) 3 (E) 18

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9. Consider the curve $f(x)=x^2$ from $x_1=3$ to $x_2=6$. Find c, where $x_1 \le c \le x_2$, such that $f(c)\left(x_2-x_1\right)=\int_{x_1}^{x_2} f(x)\,dx$.

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14. Given $f(x)=\sqrt{x}$, a value for c such that $f(b)=f'(c)(b-a)+f(a)$ for $a=4$ and $b=9$ is

(A) $3\frac{7}{8}$ (B) $4\frac{3}{4}$ (C) $5\frac{3}{8}$ (D) $6\frac{1}{4}$ (E) $7\frac{1}{2}$

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13. If $f(x)=x^3$, $a=0$, $b=6$, and $f(b)-f(a)=(b-a)f'(c)$, then the value of c is

(A) 2 (B) $2\sqrt{2}$ (C) $2\sqrt{3}$ (D) $2\sqrt{6}$
(E) none of the above

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9. Given $f(x)=\frac{x}{2}+\frac{2}{x}$ over the closed interval $[a, b]$, a value for c such that $f(b)=f(a)+f'(c)(b-a)$ for $a=\frac{1}{2}$ and $b=4$ is

(A) $-\sqrt{2}$ (B) $-\frac{1}{2}$ (C) $\frac{1}{2}$ (D) $\sqrt{2}$
(E) none of the above

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12. Consider the curve $f(x)=x^2$ from $x_1=0$ to $x_2=3$. Find c where $x_1 \le c \le x_2$ such that $f(c)\left(x_2-x_1\right)=\int_{x_1}^{x_2} f(x)\,dx$.
