

8.2 Integration by Parts (Part I)

Example:

$$\frac{d}{dx}(x \sin x) = x \cos x + \sin x \qquad \frac{d}{dx}(x \sin x) - \sin x = x \cos x$$

$$\int \left[\frac{d}{dx}(x \sin x) - \sin x \right] dx = \int x \cos x dx$$

$$x \sin x - \int \sin x dx = \int x \cos x dx$$

$$x \sin x + \cos x = \int x \cos x dx$$

$$\int x \cos x dx = x \sin x + \cos x + c$$

Proof: if u and v are both differentiable functions

From the product rule for derivatives $\frac{d}{dx}(u \cdot v) = u \cdot \frac{dv}{dx} + v \cdot \frac{du}{dx}$

then $\frac{d}{dx}(u \cdot v) - v \cdot \frac{du}{dx} = u \cdot \frac{dv}{dx}$

integrate both sides $\int \left[\frac{d}{dx}(u \cdot v) - v \cdot \frac{du}{dx} \right] dx = \int u \cdot \frac{dv}{dx} dx$

and $u \cdot v - \int v \cdot du = \int u \cdot dv$

reflecting $\int u dv = u \cdot v - \int v du$

Evaluate each of the following indefinite integrals using integration by parts.

$$\int \ln x dx$$

$$2. \int x \ln x dx$$

$$3. \int x \sin x dx$$

$$4. \int x e^x dx$$

$$5. \int x^2 e^x dx$$

$$6. \int x^3 e^{-2x} dx$$

$$7. \int x^2 \cos 2x dx$$

$$8. \int x^2 \ln x dx$$

$$9. \int \tan^{-1} x dx$$

$$10. \int \sin^{-1} x dx$$

$$11. \int x^3 e^{x^2} dx$$

$$12. \int e^x \sin x dx$$

8.2 Integration by Parts (Part II)

1. $\int \cot x \ln(\sin x) dx$

2. Evaluate $\int_0^1 x f''(x) dx$ if $f(0) = 6$, $f(1) = 5$, and $f'(1) = 2$

3. $\int \frac{\ln x}{x^2} dx$

4. $\int \sin 2x \ln(\sin x) dx$

5. Given the region bounded by the x -axis, $f(x) = \frac{\ln x - 1}{x^2}$ over the interval $e \leq x \leq e^3$,

Set up the integrals to:

a.) Find the area of the region

b.) Find volume of the solid of revolution if the region is revolved around the y -axis.

6. Let f be a differentiable function such that f'' is continuous and f and f' have the values given in the table.

x	1	2	3	4
$f(x)$	2	-2	-3	-7
$f'(x)$	-1	1	2	5

Evaluate a.) $\int_1^2 x f'(x^2) dx$ b.) $\int_1^3 x f''(x) dx$ c.) $\int_1^2 x^3 f''(x^2) dx$

Answers:

1. $\frac{\ln^2(\sin x)}{2} + c$ 2. 3.0 3. $\frac{-\ln x - 1}{x} + c$ 4. $\sin^2 x \ln(\sin x) - \frac{\sin^2 x}{2} + c$

5a) $\int_e^{e^3} \frac{\ln x - 1}{x^2} dx$ b) $2\pi \int_e^{e^3} \frac{\ln x - 1}{x} dx$ 6. a) -4.5 b) 12 c) 15

8.5 Integrating Rational Expressions Using Partial Fractions

1. $\int \frac{3}{x-4} dx$

2. $\int \frac{5}{4x-2} dx$

3. $\int \frac{3x}{x-4} dx$

4. $\int \frac{2x^2}{x^2+1} dx$

5. $\int \frac{x^3 - 4x}{x^2 + 1} dx$

6. $\int \frac{2x^2 + x - 1}{x^2} dx$

7. $\int \frac{x+3}{(x-2)(3x-4)} dx$

8. $\int \frac{2}{x^2-9} dx$

9. $\int \frac{3x+2}{x^3-4x^2-5x} dx$

10. $\int \frac{2x^3 - x^2 + 1}{x^3 - 9x} dx$

11. $\int_0^1 \frac{x^3 - 4x - 10}{x^2 - x - 6} dx$

12. $\int \frac{9x^2}{x^2-1} dx$