A.P. Calculus 2 - Practice Exam.

For each series, you are to determine whether it is convergent or diverergent. You may do the problem using any test you wish, but you must identify the test you use and then justify your answer (example: p-series, $n > 1$ or geometric series, $r > 1$ or limit comparison to $\frac{1}{n}$ or ratio test, $\lim_{n\to\infty} < 1$, etc.

The tests are: nth term, geometric, telescoping, p-series, alternating series, integral, root, ratio, direct comparison, limit comparison.

1. $\sum_{n=1}^{\infty} \frac{n+3}{2n-5}$

$\lim_{n\to\infty} \frac{n+3}{2n-5} = \frac{1}{2}$

Diverge
n^{th} term

2. $\sum_{n=1}^{\infty} \frac{n}{n^3+1}$

$\lim_{n\to\infty} \frac{\frac{n}{n^3+1}}{\frac{1}{n^2}} = \lim_{n\to\infty} \frac{n^3}{n^3+1} = 1$

Converge
Limit Comparison

3. $\sum_{n=1}^{\infty} \frac{n}{\sqrt{n^2+3}}$

$\lim_{n\to\infty} \frac{n}{\sqrt{n^2+3}} = 1$

n^{th} term
Diverges

4. $\sum_{n=1}^{\infty} \frac{3^n}{n^3}$

$\lim_{n\to\infty} \left| \frac{3^{n+1}}{(n+1)^3} \cdot \frac{n^3}{3^n} \right| = 3$

Diverge
Ratio

5. $\sum_{n=1}^{\infty} (1.002)^n$

Diverge
geometric
$r > 1$

6. $\sum_{n=1}^{\infty} \left(\frac{2n}{3n+1}\right)^{2n}$

$\lim_{n\to\infty} \sqrt[n]{\left(\frac{2n}{3n+1}\right)^{2n}} = \lim_{n\to\infty} \left(\frac{2n}{3n+1}\right)^2$

$= \frac{4n^2}{(3n+1)^2} = \frac{4}{9}$ ✓

Converge Root Test

7. $\sum_{n=1}^{\infty} (-1)^n \frac{n}{3n-1}$

$\lim_{n\to\infty} \frac{n}{3n-1} = \frac{1}{3}$

Diverge
Alternating Series

8. $\sum_{n=1}^{\infty} \frac{4^n}{n!}$

$\lim_{n\to\infty} \frac{4^{n+1}}{(n+1)!} \cdot \frac{n!}{4^n}$

$\lim_{n\to\infty} \frac{4}{n+1} = 0$ ✓

converge by Ratio

9. $\sum_{n=1}^{\infty} \frac{1}{n^{7/6}}$

p-series $p > 1$
converge

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$1, -1, 1$

10. $\sum_{n=1}^{\infty} \frac{1}{n} \sin \frac{(2n-1)\pi}{2}$ alt.

$\lim_{n\to\infty} \frac{1}{n} = 0$

$\frac{1}{n}$ is decreasing

Converge Alternating

11. $\sum_{n=2}^{\infty} \frac{1}{n(\ln n)^2}$

$\int^{\infty} (\ln x)^{-2} \frac{1}{x}\, dx$ $\quad u = \ln x$, $du = \frac{1}{x} dx$

$\int_{\ln 2}^{\ln \infty} u^{-2}\, du$

$-\frac{1}{u}\Big|_{\ln 2}^{\ln \infty} = \frac{1}{\infty} + \frac{1}{\ln 2}$, 0

Converge Integral

12. $\sum_{n=1}^{\infty} \frac{4}{2n-1}$

$\lim_{n\to\infty} \left| \frac{4}{2n-1} \cdot \frac{n}{1} \right| = 2$

Diverge lim comp.

13. $\sum_{n=1}^{\infty} \frac{6}{2^n} = \sum 6\left(\frac{1}{2}\right)^n$

Converge geometric $r<1$

14. $\sum_{n=1}^{\infty} \left(\frac{1}{n} - \frac{1}{n+2}\right)$

$1 - \frac{1}{3} + \frac{1}{2} - \frac{1}{4} + \frac{1}{3} - \frac{1}{5} + \frac{1}{4} - \frac{1}{6} \ldots \frac{1}{n}$

$\frac{3}{2}$ $\quad 0$

converge Telescoping

15. $\sum_{n=1}^{\infty} \frac{4^n}{5^n+1}$

$\frac{4^n}{5^n+1} < \frac{4^n}{5^n}$ conv. geo

Converge Direct Comp.

16. $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{\sqrt{n}}$ Alt.

$\lim_{n\to\infty} \frac{1}{\sqrt{n}} = 0$

$\frac{1}{\sqrt{n}}$ decreasing

converge Alt. series

17. $\sum_{n=1}^{\infty} \frac{1}{n^2+4}$

$\frac{1}{n^2+4} < \frac{1}{n^2}$ conv. p-series

conv. Direct Comp

Integral or lim comp

18. $\sum_{n=1}^{\infty} n\left(\frac{3}{4}\right)^n$

$\lim_{n\to\infty} \frac{(n+1)\left(\frac{3}{4}\right)^{n+1}}{n\left(\frac{3}{4}\right)^{n}} = \frac{3}{4}$

conv. Ratio Test

19. $\sum_{n=1}^{\infty} (.9)^n$

converge geometric $r<1$

20. $\sum_{n=1}^{\infty} \frac{n!}{n2^n}$

$\lim_{n\to\infty} \frac{(n+1)!}{(n+1)2^{n+1}} \cdot \frac{n(2^n)}{n!}$

$\lim_{n\to\infty} \frac{n}{2} = \infty > 1$

Diverge Ratio test

21. $\sum_{n=1}^{\infty} \frac{1000}{\sqrt[7]{n}}$

diverge p-series $p<1$