6.3 U- Substitution

1. $\int (2x+5)(x^2+5x)^7\,dx$
2. $\int (3-x)^{10}\,dx$
3. $\int e^{5x+2}\,dx$
4. $\int 4\cos(3x)\,dx$
5. $\int 5\sec 4x \tan 4x\,dx$
6. $\int \sqrt{7x+9}\,dx$
7. $\int \frac{\sin(\ln x)}{x}\,dx$
8. $\int \frac{3x+6}{x^2+4x-3}\,dx$
9. $\int \frac{x^3}{(1+x^4)^{1/3}}\,dx$
10. $\int \frac{3}{x\ln x}\,dx$
11. $\int \frac{\cos(5x)}{e^{\sin(5x)}}\,dx$
12. $\int (\sec^2 x)\sqrt{5+\tan x}\,dx$
13. $\int (x+3)(x-1)^5\,dx$
14. $\int \frac{x+5}{2x+3}\,dx$
15. $\int \frac{x^2+4}{x+2}\,dx$
16. $\int x\sqrt{4-x}\,dx$
17. $\int \frac{\cos^2 x}{1+\sin x}\,dx$
18. $\int \tan 5x\,dx$
19. $\int (2+\tan x)^2\,dx$
20. $\int (\csc 3x+\cot 3x)^2\,dx$

1. $\int_{-1}^{0} \sqrt{5x+9}\,dx$
2. $\int_{0}^{1} x\left(2x^2-1\right)^3\,dx$
3. $\int_{0}^{\frac{\pi}{2}} \cos 2x\,dx$
4. $\int_{\frac{\pi}{6}}^{\frac{\pi}{2}} \frac{\cos x}{\sin^2 x}\,dx$
5. $\int_{\ln 2}^{\ln 4} e^{3x}\,dx$
6. $\int_{0}^{2} \frac{x}{(x^2+1)^2}\,dx$
7. $\int_{0}^{1} \frac{e^x}{e^{2x}+1}\,dx$
8. $\int_{0}^{1} \frac{x^2}{x+1}\,dx$
9. $\int_{4}^{5} x^2\sqrt{x-4}\,dx$
10. $\int_{-2}^{1} (x-1)\sqrt{x+3}\,dx$

Let $F(x)=\int_0^x (t^3-2t^2)\,dt$. Find:

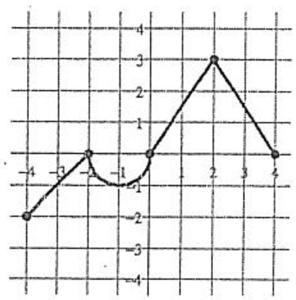
a.) $F(1)$, $F'(1)$, $F''(1)$

b.) Values of x where there are horizontal tangent lines to $F(x)$.

c.) Find an equation of the line tangent to the graph of $F(x)$ at $x=1$.

d.) Suppose that the tangent line in part (b) is used to compute an estimate of $F(1.02)$. Is this estimate too large or too small? Justify your answer.

Let f be the continuous function graphed below and let $F(x)=\int_{-2}^{x} f(t)\,dt$.



a.) Find $F(2)$.

b.) Find $F'(-3)$.

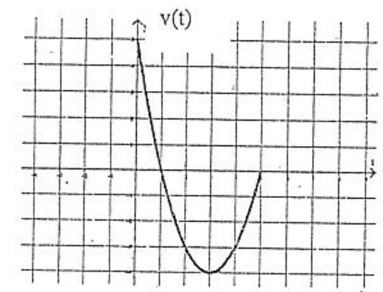
c.) Find all values of x in the open interval $(-4,4)$ at which F attains a relative minimum. Justify your answer.

d.) Find the absolute maximum value of F on the closed interval $[-4,4]$. Justify your answer.

e.) Is there a point of inflection on the graph of F at $x = -2$? Justify your answer.

f.) Over what intervals is the graph of F concave down? Justify your answer.

1. The graph below shows the **velocity** $v(t)$, (in feet/sec), of a particle moving vertically along a line for any time t (in seconds) for $0 \le t \le 5$. The particle is at the origin at t = 0.



a.) What direction is the particle moving at $t = 4$ seconds?
b.) What is the particle's instantaneous velocity at $t = 2$?
c.) What is the particle's speed at $t = 2$?
d.) Estimate the particle's average velocity from $t = 0$ to $t = 4$?
e.) When is the particle moving up, down, at rest?
f.) When is the particle's velocity increasing ,decreasing
g.) When is the particle's speed increasing , decreasing
h.) Estimate the net displacement of the particle from t = 0 to t = 5.
i.) Estimate the total distance traveled by the particle from t = (to t = 5.

2. A particle, starting at $x = 2$ at time $t = 0$, is moving horizontally v velocity $v(t) = -t+2$, for $t \ge 0$.

a.) Find the location of the particle, $x(t)$, at any time t.

b.) Find the displacement of the particle over the time interval $0 \le t \le 4$.

c.) Find the total distance traveled by the particle over the time interval $0 \le t \le 4$.

3. A particle is moving horizontally with acceleration $a(t) = 2t - 4$, for $t \geq 0$. At time $t = 0$, $v(0)=3$ and $x(0)=1$.

a.) At what time is the particle at rest?

b.) Find the location of the particle, $x(t)$, at any time t.

c.) Find the displacement of the particle over the time interval $0 \leq t \leq 3$.

d.) Find the total distance traveled by the particle over the time interval $0 \leq t \leq 3$.

4. Find the total area between the curve $f(x) = \cos x$ and the x-axis for $0 \leq t \leq 2\pi$

5. Derive the motion of free-fall objects

If a rocket propelled from the ground level directly upwards with initial velocity of 64 ft./sec, how high will it travel?

If a penny is dropped from a bridge and the splash is heard 2 seconds later, how high is the bridge?

Determine what each of the following quantities represents in context of the information given.

6. If $c(t)$ is the rate of change of the number of cars per hour passing over a bridge after midnight what does $\int_0^6 c(t)\,dt$ represent?

7. If $w(t)$ is the rate of change of the amount of water in gallons per minute filling a tank, what does $\int_0^5 w(t)\,dt$ represent?

8. If $r(t)$ is the rate of growth of a child in inches per year then $\int_1^2 r(t)\,dt =$

9. If $s(t)$ is the rate of sewage spilling out of a pipe in gallons per second then $\int_{10}^{20} s(t)\,dt = ?$

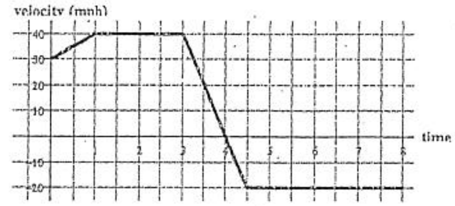
10. If $e(t)$ is the rate of change of the number of guests entering Disneyland after opening (8 AM), in hundreds per hours and if $l(t)$ is the rate of change of the number of guests leaving after opening (8 AM), in hundreds per hour what does each of the following quantities represent?

$e(4)$ $\qquad \int_0^4 e(t)\,dt$ $\qquad \dfrac{\int_0^4 e(t)\,dt}{4-0}$ $\qquad \int_0^4 l(t)\,dt$ $\qquad \int_0^4 \left(e(t) - l(t)\right)dt$

11. If $n(t)$ is the rate of change of the number of mosquitoes hatching in thousands per day and if $d(t)$ is the rate of change in the number of mosquitoes dying in thousands per day what does each of the following quantities represent?

$n(30)$ $\qquad \int_0^{30} n(t)\,dt$ $\qquad \int_0^{30} d(t)\,dt$ $\qquad \dfrac{\int_0^{30} n(t)\,dt}{30-0}$ $\qquad \int_0^{30} \left(n(t) - d(t)\right)dt$

1. A car is traveling along a straight highway **toward** Boston. At time $t = 0$, the car is 180 miles from Boston. Below is a graph of the car's velocity, plotted against time t, in hours.



a.) Find $\int_0^3 v(t)\,dt$. Explain the meaning of $\int_0^3 v(t)\,dt$.

b.) Is the car speeding up or slowing down at $t = 4.2$ hours?

c.) At what time does the car change directions? Justify your answer.

d.) When was the car closest to Boston?

e.) Did the car reach Boston during the 8 hour trip?

2. A car travels on a straight track. During the time interval $0 \le t \le 80$ seconds, the car's velocity v, measured in feet per second, is a continuous function. The table below shows selected values of this function.

t (seconds)	0	10	20	30	40	50	60	70	80
v(t) (feet per second)	-20	-30	-20	-14	-10	0	10	14	20

a.) Estimate the acceleration of the car at time $t = 20$. Show the work that leads to your answer. Indicate units of measurement.

b.) Using correct units, explain the meaning of $\int_{10}^{70} |v(t)|\,dt$.

Use the left endpoint approximation with three equal subdivisions to approximate $\int_{10}^{70} |v(t)|\,dt$.

c.) Find the exact value of $\int_{30}^{60} a(t)\,dt$. Using correct units, describe what the quantity represents

3. Water is being pumped into an underground tank at a consta rate of 8 gallons per minute. The rate at which water is leak out of the tank is continuously increasing. Suppose the rate which water is leaking out of the tank (in gallons per minute given by the function $L(t) = \sqrt{t+1}$, for $0 \le t \le 120$ minutes. $t = 0$ the tank contains 30 gallons of water.

a.) Find the average rate at which water is leaking out of t tank over the first 3 minutes. Indicate units of measure

b.) Find $\int_0^3 L(t)\,dt$. Using correct units, explain the meaning $\int_0^3 L(t)\,dt$ in terms of water leakage.

c.) Write an expression for $v(t)$, the total number of gallo of water in the tank at time t.

d.) At what time t, for $0 \le t \le 120$, is the amount of water i the tank a maximum? Justify your answer.

4. A particle moves on the x-axis so that its velocity at any time $t \ge 0$ is given by $v(t) = 12t^2 - 36t + 15$. At $t = 1$, the particle is at the origin.

a.) Find the position $x(t)$ of the particle at any time $t \ge 0$.

b.) Find all values of t for which the particle is at rest.

c.) Find the maximum velocity of the particle for $0 \le t \le 2$.

d.) Find the total distance traveled by the particle from $t = 0$ to $t = 2$.

e.) What is the furthest distance away from the origin from $t = 0$ to $t = 2$.