u-Substitution - Classwork

When you take derivatives of more complex expressions, you frequently have to use the chain rule to differentiate. The integration equivalent of the chain rule is called *u-substitution*. *u-substitution* allows you integrate expressions which do not appear integratable.

1) $\int x(x^2-1)^5\,dx$ Set up a $u = x^2-1$ Find $\frac{du}{dx} = 2x$. Solve for $du = 2x\,dx$

$\frac{1}{2}\int 2x(x^2-1)^5\,dx$ You need to manufacture your du in the original expression. So you will have to multiply by ______ on the inside and thus multiply by ________ on the outside.

$\frac{1}{2}\int u^5\,du = \frac{u^6}{12} = \frac{(x^2+1)^6}{12} + C$ Now change everything to u. Now integrate in terms of u.

Finally, change back to the variaable x and add C.

2) $\int (3x-2)^4\,dx$

$\frac{1}{3}\int u^4\,du = \frac{u^5}{15}$ $u = 3x-2$

$\frac{(3x-2)^5}{15} + C$ $du = 3\,dx$

3) $\int \sqrt{5x-2}\,dx$

$\frac{1}{5}\int u^{\frac{1}{2}}\,du = \frac{2u^{\frac{3}{2}}}{15}$ $u = 5x-2$

$\frac{2(5x-2)^{\frac{3}{2}}}{15} + C$ $du = 5\,dx$

4) $\int 4(6x-1)^{2/3}\,dx$

$\frac{4}{6}\int u^{\frac{2}{3}}\,du = \frac{4}{6}\cdot\frac{3u^{\frac{5}{3}}}{5}$ $u = 6x-1$

$\frac{2(6x-1)^{\frac{5}{3}}}{5} + C$ $du = 6\,dx$

5) $\int x\sqrt{x^2-2}\,dx$

$\frac{1}{2}\int u^{\frac{1}{2}}\,du = \frac{1}{2}\cdot\frac{2u^{\frac{3}{2}}}{3}$ $u = x^2-2$

$\frac{(x^2-2)^{\frac{3}{2}}}{3} + C$ $du = 2x\,dx$

6) $\int x^2\sqrt{1-4x^3}\,dx$

$-\frac{1}{12}\int u^{\frac{1}{2}}\,du = \frac{-1}{12}\cdot\frac{2u^{\frac{3}{2}}}{3}$ $u = 1-4x^3$

$\frac{-(1-4x^3)^{\frac{3}{2}}}{18} + C$ $du = -12x^2\,dx$

7) $\int \frac{x}{\sqrt[3]{2x^2-1}}\,dx$

$\frac{1}{4}\int u^{\frac{-1}{3}}\,du = \frac{1}{4}\cdot\frac{3u^{\frac{2}{3}}}{2}$ $u = 2x^2-1$

$\frac{3(2x^2-1)^{\frac{2}{3}}}{8} + C$ $du = 4x\,dx$

8) $\int x^{1/2}\left(x^{3/2}+2\right)^9\,dx$

$\frac{2}{3}\int u^9\,du = \frac{2}{3}\cdot\frac{u^{10}}{10}$ $u = x^{3/2}+2$

$\frac{\left(x^{3/2}+2\right)^{10}}{15} + C$ $du = \frac{3}{2}x^{1/2}\,dx$

9) $\int (x+2)\sqrt{x^2+4x-3}\,dx$

$\frac{1}{2}\int u^{\frac{1}{2}}\,du = \frac{1}{2}\cdot\frac{2u^{\frac{3}{2}}}{3}$ $u = x^2+4x-3$

$\frac{(x^2+4x-3)^{\frac{3}{2}}}{3} + C$ $du = 2(x+2)\,dx$

10) $\int (x+2)\sqrt{x-4}\,dx$

$\int (u+6)u^{\frac{1}{2}}\,du$

$\int \left(u^{\frac{3}{2}}+6u^{\frac{1}{2}}\right)du$ $\qquad u=x-4 \qquad x=u+4$

$\frac{2}{5}u^{\frac{5}{2}}+6\cdot\frac{2}{3}u^{\frac{3}{2}}$ $\qquad du=\;dx$

$\frac{2}{5}(x-4)^{\frac{5}{2}}+4(x-4)^{\frac{3}{2}}+C$

11) $\int \frac{x-5}{\sqrt{x-6}}\,dx$

$\int (u+1)u^{\frac{-1}{2}}\,du$

$\int \left(u^{\frac{1}{2}}+u^{\frac{-1}{2}}\right)du$ $\qquad u=x-6 \qquad x=u+6$

$\frac{2}{3}u^{\frac{3}{2}}+2u^{\frac{1}{2}}$ $\qquad du=\;dx$

$\frac{2}{3}(x-6)^{\frac{3}{2}}+2(x-6)^{\frac{1}{2}}+C$

12) $\int \frac{x^2}{\sqrt{x+1}}\,dx$

$\int \left(u^{\frac{3}{2}}-2u^{\frac{1}{2}}+u^{-\frac{1}{2}}\right)du$

$\frac{2}{5}u^{\frac{5}{2}}-\frac{4}{3}u^{\frac{3}{2}}+2u^{\frac{1}{2}}$ $\qquad u=x+1 \qquad x=u-1$

$\frac{2}{5}(x+1)^{\frac{5}{2}}-\frac{4}{3}(x+1)^{\frac{3}{2}}+2(x+1)^{\frac{1}{2}}+C$ $\qquad du=\;dx$

13) $\int \cos 4x\,dx$

$\frac{1}{4}\int \cos u\,du=\frac{1}{4}\cdot\sin u$ $\qquad u=4x$

$\frac{\sin 4x}{4}+C$ $\qquad du=4\,dx$

14) $\int 3\sin(1-3x)\,dx$

$\frac{3}{-3}\int \sin u\,du=-(-\cos u)$ $\qquad u=1-3x$

$\cos(1-3x)+C$ $\qquad du=-3\,dx$

15) $\int \sin^3 x\cos x\,dx$

$\int u^3\,du=\frac{u^4}{4}$ $\qquad u=\sin x$

$\frac{\sin^4 x}{4}+C$ $\qquad du=\cos x\,dx$

16) $\int \tan 10x\sec 10x\,dx$

$\frac{1}{10}\int \tan u\sec u\,du=\sec u$ $\qquad u=10x$

$\frac{\sec 10x}{10}+C$ $\qquad du=10\,dx$

17) $\int \tan^2 x\sec^2 x\,dx$

$\int u^2\,du=\frac{u^3}{3}$ $\qquad u=\tan x$

$\frac{\tan^3 x}{3}+C$ $\qquad du=\sec^2 x\,dx$

18) $\int \sin x\sqrt{\cos x}\,dx$

$-\int u^{\frac{1}{2}}\,du=\frac{-2u^{\frac{3}{2}}}{3}$ $\qquad u=\cos x$

$\frac{-2(\cos x)^{\frac{3}{2}}}{3}+C$ $\qquad du=-\sin x\,dx$

19) $\int \frac{\cos x}{\sqrt{1-\sin x}}\,dx$

$-\int u^{\frac{-1}{2}}\,du=-2u^{\frac{1}{2}}$ $\qquad u=1-\sin x$

$-2\sqrt{1-\sin x}+C$ $\qquad du=-\cos x\,dx$

u-Substitution - Homework

1. $\int \sqrt{x-2}\,dx$

$$\frac{2(x-2)^{\frac{3}{2}}}{3}+C$$

2. $\int (2x+3)^{11}\,dx$

$$\frac{(2x+3)^{12}}{24}+C$$

3. $\int \sqrt{5x-1}\,dx$

$$\frac{2(5x-1)^{\frac{3}{2}}}{15}+C$$

4. $\int \sqrt[3]{6x+1}\,dx$

$$\frac{(6x+1)^{\frac{4}{3}}}{8}+C$$

5. $\int 5(3-4x)^{2/3}\,dx$

$$\frac{-3(3-4x)^{\frac{5}{3}}}{4}+C$$

6. $\int \frac{dx}{(8x-1)^3}$

$$\frac{-1}{16(8x-1)^2}+C$$

7. $\int x(x^2+2)^6\,dx$

$$\frac{(x^2+2)^7}{14}+C$$

8. $\int 6x^2\sqrt{3x^3-1}\,dx$

$$\frac{4(3x^3-1)^{\frac{3}{2}}}{9}+C$$

9. $\int \left(1+\frac{1}{x}\right)^3\left(\frac{1}{x^2}\right)dx$

$$\frac{-\left(1+\frac{1}{x}\right)^4}{4}+C$$

10. $\int x^{1/3}\left(x^{4/3}+9\right)^8\,dx$

$$\frac{\left(x^{4/3}+9\right)^4}{12}+C$$

11. $\frac{2}{3}\int \sqrt{4-\frac{3}{5}x}\,dx$

$$\frac{-20\left(4-\frac{3}{5}x\right)^{\frac{3}{2}}}{27}+C$$

12. $\int (3x+15)\sqrt{x^2+10x+4}\,dx$

$$\left(x^2+10x+4\right)^{\frac{3}{2}}+C$$

13. $\int (x+2)\sqrt{x-2}\,dx$

$$\frac{2(x-2)^{\frac{5}{2}}}{5}+\frac{8(x-2)^{\frac{3}{2}}}{3}+C$$

14. $\int \frac{x^2}{\sqrt{x-4}}\,dx$

$$\frac{2(x-4)^{\frac{5}{2}}}{5}+\frac{16(x-4)^{\frac{3}{2}}}{3}+32(x-4)^{\frac{1}{2}}+C$$

15. $\int \sin 5x\,dx$

$$\frac{-\cos 5x}{5}+C$$

16. $\int \cos\frac{x}{2}\,dx$

$$2\sin\frac{x}{2}+C$$

17. $\int \frac{1}{3}\sec^2 8x\,dx$

$$\frac{\tan 8x}{24}+C$$

18. $\int \sin 4x\cos 4x\,dx$

$$\frac{\sin^2 4x}{8}+C \text{ or } \frac{-\cos^2 4x}{8}+C$$

19. $\int \cos^3 x\sin x\,dx$

$$\frac{-\cos^4 x}{4}+C$$

20. $\int \tan x\sec^2 x\,dx$

$$\frac{\tan^2 x}{2}+C$$

21. $\int \sqrt{\cos 6x}\sin 6x\,dx$

$$\frac{-(\cos(6x))^{\frac{3}{2}}}{9}+C$$

22. $\int \frac{\sin x}{(4-\cos x)^3}\,dx$

$$\frac{-1}{2(4-\cos x)^2}+C$$