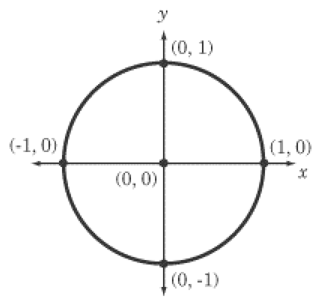
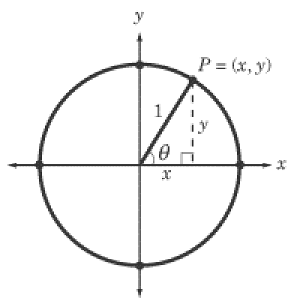
The Unit Circle

The **unit circle** is a circle whose center is the origin and whose radius is 1. It is defined by equation $x^2 + y^2 = 1$.



The most useful and interesting property of the unit circle is that the coordinates of a given point on the circle can be found using only the measure of the angle.



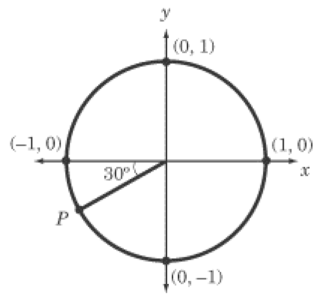
Any radius of the unit circle is the hypotenuse of a right triangle that has a (horizontal) leg of length cos θ and a (vertical) leg of length sin θ. The angle θ is defined as the radius measured in standard position. These relationships are easy to see using the trigonometric functions:

$$\sin\theta = \frac{y}{1} = y$$
$$\cos\theta = \frac{x}{1} = x$$
$$\tan\theta = \frac{y}{x}$$

As you can see, because the radius of the unit circle is 1, the trigonometric functions sine and cosine are simplified: sin θ = y and cos θ = x. This means that another way to write the coordinates of a point (x, y) on the unit circle is (cos θ, sin θ), where θ is the measure of the angle in standard position whose terminal side contains the point.

Here's an example of a typical Math IC question that tests this principle:

What are the coordinates of the point P pictured below?



Point *P* is the endpoint of a radius of the unit circle that forms a 30º angle with the negative *x*-axis. This means that an angle of 210º in standard position would terminate in the same position. So, the coordinates of the point are (cos 210º, sin 210º) = $(-\sqrt{3}/_2, -^1/_2)$. Both coordinates must be negative, since the point is in the third quadrant.

Range

The unit circle also provides a lot of information about the range of trigonometric functions and the values of the functions at certain angles.

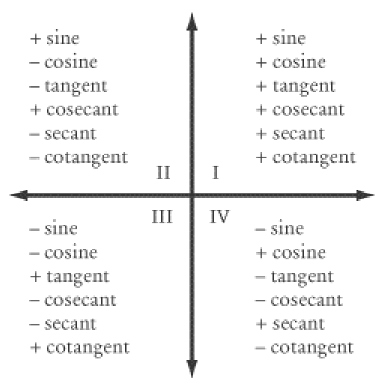
For example, because the unit circle has a radius of one and its points are all of the form (cos θ, sin θ), we know that:

$$-1 < \sin\theta < 1 \text{ and } -1 < \cos\theta < 1$$

Tangent ranges from –∞ to ∞, but it is undefined at every angle whose cosine is 0. Can you guess why? Look at the formula of tan θ = $^{\sin\theta}/_{\cos\theta}$. If cos θ = 0, then division by 0 occurs, and so the quotient, tan θ, is undefined.

The Unit Circle and Important Angles

Using the unit circle makes it easy to find the values of trigonometric functions at quadrantal angles. For example, a 90º rotation from the positive *x*-axis puts you on the positive *y*-axis, which intersects the unit circle at the point (0, 1). From this, you know that (cos 90º, sin 90º) = (0, 1). Here is a graph of the values of all three trigonometric functions at each quadrantal angle:



There are a few other common angles besides the quadrantal angles whose trigonometric function values you should already know. Listed below are the values of sine, cosine, and tangent taken at 30º, 45º, and 60º. You might recognize some of these values from the section on special triangles.

	$30° = \frac{\pi}{6}$ rad	$45° = \frac{\pi}{4}$ rad	$60° = \frac{\pi}{3}$ rad
$\sin\theta$	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{3}{2}$
$\cos\theta$	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$
$\tan\theta$	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$

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