6. In order to monitor the populations of birds of a particular species on two islands, the following procedure was implemented.

Researchers captured an initial sample of 200 birds of the species on Island A; they attached leg bands to each of the birds, and then released the birds. Similarly, a sample of 250 birds of the same species on Island B was captured, banded, and released. Sufficient time was allowed for the birds to return to their normal routine and location.

Subsequent samples of birds of the species of interest were then taken from each island. The number of birds captured and the number of birds with leg bands were recorded. The results are summarized in the following table.

<table>
<thead>
<tr>
<th></th>
<th>Island A</th>
<th>Island B</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number Captured in Subsequent Sample</td>
<td>180</td>
<td>220</td>
</tr>
<tr>
<td>Number with Leg Bands in Subsequent Sample</td>
<td>12</td>
<td>35</td>
</tr>
</tbody>
</table>

Assume that both the initial sample and the subsequent samples that were taken on each island can be regarded as random samples from the population of birds of this species.

(a) Do the data from the subsequent samples indicate that there is a difference in proportions of the banded birds on these two islands? Give statistical evidence to support your answer.

(b) Researchers can estimate the total number of birds of this species on an island by using information on the number of birds in the initial sample and the proportion of banded birds in the subsequent sample. Use this information to estimate the total number of birds of this species on Island A. Show your work.

(c) The analyses in parts (a) and (b) assume that the samples of birds captured in both the initial and subsequent samples can be regarded as random samples of the population of birds of this species that live on the respective islands. This is a common assumption made by wildlife researchers. Describe two concerns that should be addressed before making this assumption.
Solution

Part (a):

Step 1: States a correct pair of hypotheses

Let

\[ p_A = \text{proportion of banded birds on island A} \]
\[ p_B = \text{proportion of banded birds on island B} \]

\[ H_0 : p_A - p_B = 0 \quad \text{OR} \quad H_0 : p_A = p_B \]
\[ H_0 : p_A - p_B \neq 0 \quad H_0 : p_A \neq p_B \]

Step 2: Identifies a correct test (by name or by formula) and checks appropriate assumptions. Two-sample test for proportions

\[ z = \frac{\hat{p}_A - \hat{p}_B}{\sqrt{\frac{\hat{p}(1-\hat{p})}{n_1} + \frac{\hat{p}(1-\hat{p})}{n_2}}} \]

Where

\[ \hat{p} = \frac{n_A \hat{p}_A + n_B \hat{p}_B}{n_A + n_B} = \frac{12 + 25}{180 + 220} = 0.1175 \]

Assumptions: independent random samples and large sample sizes.

The problem states that it is reasonable to regard the samples as random samples. Since the samples are taken on different islands it may be reasonable to assume that they are independent. Since the expected counts are greater than 10 (or 5), the sample sizes are large enough to accurately use a z-test or a chi-squared test.

These estimates of expected counts are used to check the accuracy of normal approximations to two different binomial distributions when the standard error is not estimated from the pooled estimate of the banding probability.

These estimates of expected counts are used to assess the accuracy if a chi-squared approximation or a normal approximation to the z-statistic when the standard deviation is estimated using the pooled estimate of the banding probability.
Step 3: Correct mechanics, including the value of the test statistic and P-value (or rejection region)

\[
\hat{p}_A = \frac{12}{180} = 0.067 \quad \text{and} \quad \hat{p}_B = \frac{35}{220} = 0.159
\]

\[
\hat{p} = \frac{12 + 35}{180 + 220} = \frac{47}{400} = 0.1175
\]

\[
z = \frac{0.067 - 0.159}{\sqrt{\frac{0.1175(0.8825)}{180} + \frac{0.1175(0.8825)}{220}}} = \frac{-0.092}{0.032} = -2.875
\]

p-value = 0.00429

Step 4: Stating a correct conclusion in the context of the problem, using the result of the statistical test.
Since p-value = 0.00429 is smaller than \( \alpha = .05 \), reject the null hypothesis. There is convincing evidence that the proportions of banded birds on the two islands are not the same.

Part (b):

For island A,

\[ n_I = 200 \quad \text{where } n_I \text{ is the number of birds banded in the initial sample,} \]

\[ \hat{p}_s = \frac{12}{180} = 0.067 \quad \text{where } \hat{p}_s \text{ is the proportion banded in the subsequent sample.} \]

We expect that the proportion of banded birds in the subsequent sample is approximately equal to the proportion of the population that is banded.

\[ \hat{p}_s \approx \frac{n_I}{\text{population size}} \]

Then and the population size can be estimated by

\[ \text{estimated population size} = \frac{n_I}{\hat{p}_s} = \frac{200}{0.06667} \approx 3000 \]
Part (c):

Possible concerns are:

1. Are some birds more likely to be captured than others? If, for example, slower birds are more likely to be captured in both the initial and subsequent samples, we would tend to underestimate the population size, thinking that a larger proportion of birds has been banded than was actually the case (because the birds that were caught and banded in the initial sample were also more likely to be the ones caught in the subsequent sample).

2. It may be the case that birds that are caught and banded in the initial sample learn from the experience and are less likely to be caught as part of the second sample. This would cause us to overestimate the population size.

3. There must be enough time between the samples so that there is adequate mixing of the banded and unbanded birds.

4. By banding the birds the researchers might make them more susceptible to their predators. In order to have a reasonable estimate, we must assume that the death rate of the banded birds is the same as the death rate as the unbanded birds so the banding procedure should not harm the birds or make them more conspicuous. For example, using large fluorescent bands is not a good idea.

5. Differentiable catchability. For example, birds that spend most of their time on the nest may be much less likely to be captured than other birds, and young birds may be more likely to be captured.

6. If the time between samples is too long, births could occur in the populations. Obviously, the new arrivals will not be banded.